Cooperative Cleaners — a Study in Ant-Robotics *

Israel A. Wagner[1,2], Yaniv Altshuler[1], Vladimir Yanovski[1], and Alfred M. Bruckstein[1]

[1] Computer Science Department, Technion, Haifa 32000 Israel
{wagner, yanival, volodyan, freddy}@cs.technion.ac.il
[2] IBM Haifa Labs, MATAM, Haifa 31905 Israel
wagner@il.ibm.com

Abstract. In the world of living creatures, "simple minded" animals often cooperate to achieve common goals with amazing performance. One can consider this idea in the context of robotics, and suggest models for programming goal-oriented behavior into the members of a group of simple robots lacking global supervision. This can be done by controlling the local interactions between the robot agents, to have them jointly carry out a given mission. As a test case we analyze the problem of many simple robots cooperating to clean the dirty floor of a non-convex region in $\mathbf{Z}^2$, using the dirt on the floor as the main means of inter-robot communication.

1 Introduction

In the world of living creatures, "simple minded" animals like ants or birds cooperate to achieve common goals with surprising performance. It seems that these animals are "programmed" to interact locally in such a way that the desired global behavior is likely to emerge even if some individuals of the colony die or fail to carry out their task for some other reasons. It is suggested to consider a similar approach to coordinate a group of robots without a central supervisor, by using only local interactions between the robots. When this decentralized approach is used, much of the communication overhead (characteristic to centralized systems) is saved, the hardware of the robots can be fairly simple, and better modularity is achieved. A properly designed system should achieve reliability through redundancy.

Significant research effort has been invested during the last few years in design and simulation of multi-agent systems [Bro1,Lev1,Sen1,Ste1,Wag3,Ark1,Wag4]. Such designs are often inspired by biology (see [Ark2] or [Bro2,Ark3] for behavior based control models, [Mat1,Den1,Dro1] for flocking and dispersing models, [Ben1,Hay1] for predator-prey approach), physics (see e.g. [Che1]), or economics application (see e.g. [Ger1,Gol1,Rab1,Smi1,Tha1,Wel1,Zlo1]).

Tasks that have been of particular interest to researchers in recent years include synergetic mission planning [Ala1], fault tolerance [Par1], swarm control [Mat2], human design of mission plans [Mac1], role assignment [Sto1,Can1,Pag1], multi-robot

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path planning [Sve1,Lum1,Fer1,Yam1], formation generation [Fre1,Gor1,Ara1], traffic control [Pre1], formation keeping [Bal1,Wan1], exploration and mapping [Rek1], target tracking [Par2] and target search [LaV1].

Unfortunately the geometrical theory of such multi-agent systems is far from being satisfactory, as has been pointed out in [Bni1] and many other papers. A short discussion regarding the conceptual framework of intelligent swarms (or *swarm intelligence systems*) and the motivation behind the use of such, appears in section 2.

Our interest is focused on developing the mathematical tools necessary for the design and analysis of such systems. In a recent report [Wag2] we have shown that a number of agents can arrange themselves equidistantly in a row via a sequence of linear adjustments, based on a simple "local" interaction. The convergence of the configuration to the desired one is exponentially fast. A different way of cooperation between agents is inspired by the behavior of ant-colonies, and is described in [Bru1]. There it was proved that a sequence of ants engaged in deterministic chain pursuit will find the shortest (i.e. straight) path from the ant-hill to the food source, using only local interactions. In [Bru2] we investigate the behavior of a group of agents on $\mathbf{Z}^2$, where each ant-like agent is pursuing his predecessor, according to a discrete biased-random-walk model of pursuit on the integer grid. We proved there that the average paths of such a sequence of a(ge)nts engaged in a chain of probabilistic pursuit converge to the straight line between the origin and destination, and this happens exponentially fast.

In this paper we investigate a more complicated question concerning the behavior of many agents cooperating to explore an unknown area (for purposes of cleaning, painting, etc.), where each robot/agent can only see its near neighborhood, and the only way of inter-robot communication is by leaving traces on the common ground and sensing the traces left by other robots.

Similar works can be found at [Pol1,Koe1,Rek2,Lat1,Aca1,But1,Min1,Bat1]).

We present an algorithm for cleaning a dirty area that guarantees task completion (unless *all* robots die) and prove an upper bound on the time complexity of this algorithm. We also show simulation results of the algorithm on several types of regions. These simulations indicate that the precise time depends on the number of robots, their initial locations, and the shape of the region.

In the spirit of [Bra], we consider simple robots with only a bounded amount of memory (i.e. *finite-state-machines*). Generalizing an idea from computer graphics (see [Hen1]), we preserve the connectivity of the "dirty" region by allowing an agent to clean only so called *non-critical* points, points that do not disconnect the graph of dirty grid points. This ensures that the robots will stop only upon completing their mission. An important advantage of this approach, in addition to the simplicity of the agents, is fault-tolerance — even if almost all the agents cease to work before completion, the remaining ones will eventually complete the mission. We prove the correctness of the algorithm as well as an upper bound on its running time, and show how our algorithm can be extended for regions with obstacles.

The rest of the paper is organized as follows : in Section 3 the formal problem is defined as well as the aims and assumptions involved. The algorithm is presented in Section 4, where an analysis of its time-complexity appears in section 5. Section 6 is devoted to the problem of exploring/cleaning regions with obstacles, and is followed

by several experimental examples (section 7). We conclude the paper with a discussion and by pointing out several connections to related work, presented in section 8.

This paper is a revised and extended version of the work presented in [Wag1].

2 Swarm Intelligence — Framework and Motivation

2.1 Motivation

The experience gained due to the growing demand for robotics solutions to increasingly complex and varied challenges has dictated that a single robot is no longer the best solution for many application domains. Instead, teams of robots must coordinate intelligently for successful task execution.

In [Dia1] the authors present a detailed description of multi robots application domains, and demonstrate how multi robots systems can be more effective than a single robot in many of these domains. However, when designing such systems it should be noticed that simply increasing the number of robots assigned to a task does not necessarily improve the system's performance — multiple robots must cooperate intelligently to avoid disturbing each other's activity and achieve efficiency.

There are several key advantages to the use of such *intelligent swarm robotics*. First, such systems inherently enjoy the benefit of parallelism. In task-decomposable application domains, robot teams can accomplish a given task more quickly than a single robot, by dividing the task into sub-tasks and executing them concurrently. In certain domains, a single robot may simply be unable to accomplish the task on its own (e.g. to carry a large and heavy object). Second, decentralized systems tend to be, by their very nature, much more robust than centralized systems (or systems comprised of a single complex unit). Generally speaking, a team of robots may provide a more robust solution by introducing redundancy, and by eliminating any single point of failure. While considering the alternative of using a single sophisticated robot, we should note that even the most complex and reliable robot may suffer an unexpected malfunction, which will prevent it from completing its task. When using a multi agents system, on the other hand, even if a large number of the agents stop working for some reason, the entire group will often still be able to complete its task, albeit slower. For example, for exploring a hazardous region (such as a minefield or the surface of Mars), the benefit of redundancy and robustness offered by a multi agents system is highly noticeable.

Another advantage of the decentralized swarm approach is the ability of dynamically reallocating sub-tasks between the swarm's units, thus adapting to unexpected changes in the environment. Furthermore, since the system is decentralized, it can respond relatively quickly to such changes, due to the benefit of locality, meaning — the ability to swiftly respond to changes without the need of notifying a hierarchical "chain of command". Note that as the swarm becomes larger, this advantage becomes increasingly noticeable.

In addition to the ability of quick response to changes, the decentralized nature of such systems also increases their scalability. The scalability of multi agents systems is derived from the low overhead (both in communication and computation) such system possess. As the tasks assigned nowadays to multi agents based systems become increasingly complex, so does the importance of the high scalability of the systems.

Finally, by using heterogeneousness swarms, even more efficient systems could be designed, thanks to the utilization of "*specialists*" — agents whose physical properties enable them to perform much more efficiently in certain well defined tasks.

2.2 Simplicity of the Agents

A key principle in the notion of swarms, or multi agent robotics, is the simplicity of the agents. The notion of "simplicity" here means that the agents should be *significantly simpler* than a "single sophisticated agent", which can be constructed for the same purpose. As a result, the resources of such simple agents are assumed to be very limited, with respect to the following aspects :

- Memory resources — basic agents should be assumed to contain only $O(1)$ memory resources (i.e. the size of memory is independent of the size of the problem or the number of agents). This usually imposes many interesting limitations on the agents. For example, agents can remember only a limited history of their activities so far. Thus, protocols designed for agents with such limited memory resources are usually very simple and attempt to solve a given problem by defining some (necessarily local) basic patterns. The task is completed by a repetition of these patterns by a large number of agents.

- Sensing capabilities — defined according to the specific nature of the problem. For example, for agents moving along a 100×100 grid, a reasonable sensing radius may be 3 or 4, but certainly not 40.
- Computational resources — although agents are assumed to employ only limited computational resources, a formal definition of this constraint is hard to define. In general, most of the time-polynomial algorithms may be used.

Another aspect of swarm algorithms' simplicity is the extremely sparse use of communication. The issue of communication in multi agents systems has been extensively studied in recent years. Distinctions between implicit and explicit communication are usually made, in which implicit communication occurs as a side effect of other actions, or "through the world" (see, for example [Pag2]), whereas explicit communication is a specific act intended solely to convey information to other robots on the team.

Explicit communication can be performed in several ways, such as a short range point to point communication, a global broadcast, or by using some sort of distributed shared memory. Such memory is often treated to as a *pheromone*, used to convey small amounts of information between the agents [Yan1,Wag5,Yan2]. This approach is inspired from the coordination and communication methods used by many social insects — studies on ants (e.g. [Adl1,Grd1]) show that the pheromone based search strategies used by ants in foraging for food in unknown terrains tend to be very efficient.

In the spirit of designing a system which uses as simple agents as possible, we aspire that the agents will have as little communication capabilities as possible. With respect to the taxonomy of multi-agents discussed in [Dud1], we would be interested in using agents of the types *COM-NONE* or if necessary of type *COM-NEAR* with respect

to their communication distances, and of types *BAND-MOTION*, *BAND-LOW* or even *BAND-NONE* (if possible) with respect to their communication bandwidth.

Therefore, although a certain amount of implicit communication can hardly be avoided (due to the simple fact that by changing the environment, the agents are constantly generating some kind of implicit information), explicit communication should be strongly limited or avoided altogether, in order to fit our paradigm (note that in many works in this field, this is not the case, and communication, as well as memory, resources, are often being used in order to create complex cooperative systems).

3 The Cooperative Cleaners Problem

3.1 Problem's Definition

We shall assume that the time is discrete. Let G denote a two dimensional integer grid $\mathbf{Z}^2$, whose vertices have a binary property called '*dirtiness*'. Let $dirt_t(v)$ state the dirtiness state of the vertex v at time t, taking either the value "*on*" or "*off*". Let F_t be the dirty sub-graph of G at time t, i.e. $F_t = \{v \in G \mid dirt_t(v) = on\}$. We assume that F_0 is a single connected component. Our algorithm will preserve this property along its evolution.

Let a group of k agents that can move across the grid G (moving from a vertex to its neighbor in one time step) be placed at time t_0 on F_0.

Each agent is equipped with a sensor capable of telling the *dirtiness* status of all tiles in the digital sphere of diameter 5, which surrounds the agent. An agent is also aware of other agents which are located in these tiles, and all the agents agree on a common direction. Each tile may contain any number of agents simultaneously.

When an agent moves to a tile v, it has the possibility of cleaning this tile (i.e. causing $dirt_(v)$ to become *off*). The agents do not have any prior knowledge of the shape or size of the sub-graph F_0 except that it is a single and simply connected component.

The agents' goal is to clean G by eliminating the dirtiness entirely, meaning that the agents must ensure that :

$$\exists t_{success} \; s.t \; F_{t_{success}} = \emptyset$$

In addition, it is desired that this time span $t_{success}$ will be minimal.

In this work we impose the restriction of no central control and full 'de-centralization', i.e. all agents are identical and no explicit communication between the agents is allowed. An important advantage of this approach, in addition to the simplicity of the agents, is fault-tolerance — even if almost all the agents cease to work before completion, the remaining ones will eventually complete the mission, if possible.

3.2 Related Work

As mentioned in previous sections, the cooperative cleaners problem has significant similarity to other types of multi-agents problems, such as cooperative coverage, or cooperative de-mining problems. In recent years, much work have done, designing systems and algorithms for handling these tasks. In this process, various models and assumptions concerning the agents and their capabilities were used.

In general, most of the techniques used for the task of a distributed coverage use some sort of cellular decomposition. For example, in [Rek2] the area to be covered is divided between the agents based on their relative locations. In [But1] a different decomposition method is being used, which is analytically shown to guarantee a complete coverage of the area. Another interesting work is presented in [Aca1], discussing two methods for cooperative coverage — a probabilistic one, and a complete one (using yet again an exact cellular decomposition). All of the works mentioned above, however, rely on the assumption that the cellular decomposition of the area is possible. This in turn, require the use of memory resources, used for storing the dynamic map generated, the boundaries of the cells, etc'. As the initial size and geometric features of the area are generally not assumed to be known in advance, agents with a constant amount of memory will most likely not be able to use such algorithms. In this work, we are interested in a multi-agents system which could perform such a cooperative coverage with a use of minimal amount of memory, defined in advance, and unrelated to the size and geometry of the covered (or cleaned) area (this requirement is presented in Section 3.1). Such a system is presented in the later part of this work, and its ability to guarantee a completion of the task is shown.

Surprisingly, while many existing works concerning distributed (and decentralized) coverage present analytic proofs for the completion of the task (e.g. [Aca1,But1,Bat1]), unfortunately, most of them lack analytic bounds for the completion time (although in many cases an extensive amount of empirical of this nature is available). Although a proof for the coverage completion is an essential key in the design of a multi-agents system, analytic indicators for its efficiency should also be sought for. In this work, such results, bounding the cleaning time of the agents, are presented in Section 5. An interesting work to mention in this scope is this of [Svn1,Koe2], where a swarm of ant-like robots is used for repeatedly covering an unknown area, using a real time search method called *node counting*. By using this method, the robots are shown to be able to be able to efficiently perform such a coverage mission, and analytic bound for the coverage time are discussed.

As most coverage problems focus on achieving a complete coverage (and sometimes — in the minimal time possible), it is worth mentioning in this scope the work of [Con1], in which an interesting additional constraint is added. As this work was design for an autonomous painting system, the uniformity of the coverage was demanded (in order to maintain the same thickness of the paint layer). In addition, this work examined the problem of 3-D coverage, in contrary to most search cover systems, usually discussing the 2-D version. However, the main focus of this work was the use of an efficient single agent for this mission, instead of a cooperative multi-agents one.

4 The Cleaning Protocol

In order to solve the *Cooperative Cleaners* problem we propose a cleaning protocol, called **CLEAN**. Generalizing an idea from computer graphics (presented in [Hen1]), this protocol preserves the connectivity of the *dirty* region by preventing the agents from cleaning *critical points* — points which when cleaned disconnect the dirty region (see section 5.1). This ensures that the agents stop only upon completing their mission.

At each time step, each agent cleans its current location (assuming it is not a critical point), and moves according to a local movement rule, creating the effect of a clockwise traversal along the boundary of the dirty shape. As a result, the agents "peel" layers from the shape, while preserving its connectivity, until the shape is cleaned entirely. An example of the protocol can be seen in Figure 1.

To the basic description of the protocol given above, there are several exceptions. As the agents are equipped with a very limited sensing and storage capabilities, the basic structure of the protocol is enhanced with a set of local rules, designed for producing a pseudo-synchronization between the agents. Those rules generate *resting* and *waiting* commands for the agents, capable of delaying their actions, either within the time step (until other specific agents complete their cleaning process for this time step), or causing some agents to pause for a single time step, resuming their operation at the next time step only. A detailed description concerning the need for these additions appears in Sections 4.6 and 4.7. Note however that the rules in charge of the *resting* and *waiting* still obey the basic paradigm of this work, namely — they are local, use no prior knowledge of the problem, and do not require explicit communication between the agents, and can be implemented using a constant amount of memory resources. A schematic flowchart of the protocol is presented in Figure 3.

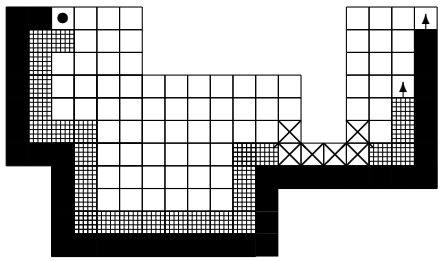


Fig. 1. An example of two agents using the **CLEAN** protocol, at time step 40. All the tiles presented were dirty at time 0. The black dot denotes the starting point of the agents. The X's mark the *critical points* which are not cleaned. The black tiles are the tiles cleaned by the first agent. The second layer of marked tiles represent the tiles cleaned by the second agent.

4.1 Cleaning Protocol — Definitions and Requirements

Let $r(t) = \big(\tau_1(t), \tau_2(t), \ldots, \tau_k(t)\big)$ denote the locations of the k agents at time t. For a tile v, let $Neighborhood(v)$ denote the dirtiness states of v and its $8Neighbors$. Let $\mathcal{M}_i$ denote some finite amount of memory contained in agent i, storing information needed for the protocol (e.g. the last move of agent i). The requested cleaning protocol is therefore a rule f such that for every agent i :

$$f\big(\tau_i(t), Neighborhood(\tau_i(t)), \mathcal{M}_i(t)\big) \in \mathcal{D}$$

where $\mathcal{D} = \{`left', `right', `up', `down'\}$.

Let ∂F denote the boundary of F. A tile is on the boundary if and only if at least one of its $8Neighbors$ is not in F, meaning :

$$\partial F = \{v \mid v \in F \ \wedge \ 8Neighbors(v) \ \cap \ (G \ \setminus \ F) \ \neq \ \emptyset\}$$

The requested rule f should meet the following goals :

- **Successful Termination** : ($\exists t_{success}$ *s.t* $F_{t_{success}} = \emptyset$).
- **Agreement on Completion** : within a finite time after completing the mission, all the agents must halt.
- **Efficiency** : the cleaning process should be efficient at time and in agents' memory resources.
- **Fault Tolerance** : if one or several stop working ("die") the rest of the agents will continue the cleaning process as efficiently as their number allows them.

4.2 The CLEAN Cleaning Protocol

The protocol is being used by each agent a_i, which at time t is located at $\tau_i(t) = (x, y)$. Follow are the definitions of several terms we use while discussing the **CLEAN** protocol. The term '*rightmost*' means :

> *Starting from $\tau_i(t-1) = (x, y)$ (which is the previous boundary tile that agent a_i had been in) scan the neighbors of (x, y) in a clockwise order until you find another boundary tile. In case $t = 0$ select the tile as instructed in Figure 2.*

The additional information needed for the protocol and its sub-routines is contained in $\mathcal{M}_i$ and $Neighborhood(x, y)$.

A schematic flowchart of the protocol, describing its major components and procedures is presented in Figure 3. The complete pseudo-code of the protocol and its sub-routines appears in Figures 4 and 5. Upon initialization of the system, the *System Initialization* procedure is called (defined in Figure 4). This procedure sets various initial values of the agents, and call the protocol's main procedure — *CLEAN* (defined in Figure 5). This procedure in turn, uses various sub-routines and functions, all defined in Figure 4.

4.3 Pivot Point

This initial location of the agents (denoted p_0) is artificially set to be critical during the execution, hence it is also guaranteed to be the last point cleaned. Completion of the mission can therefore be concluded from the fact that all (working) agents are back at p_0 with no dirty neighbors to go to, thereby reporting on completion of their individual missions. Note that this point (also called *pivot point*) is not necessary for the algorithm to work well — it just makes life easier for the user since one then knows where to find the agents after cleaning has been completed. If the agents are not placed on the pivot point at $t = 0$ and the pivot p_0 is not forced to be critical, then the location of the agents upon completion will generally not be known in advance.

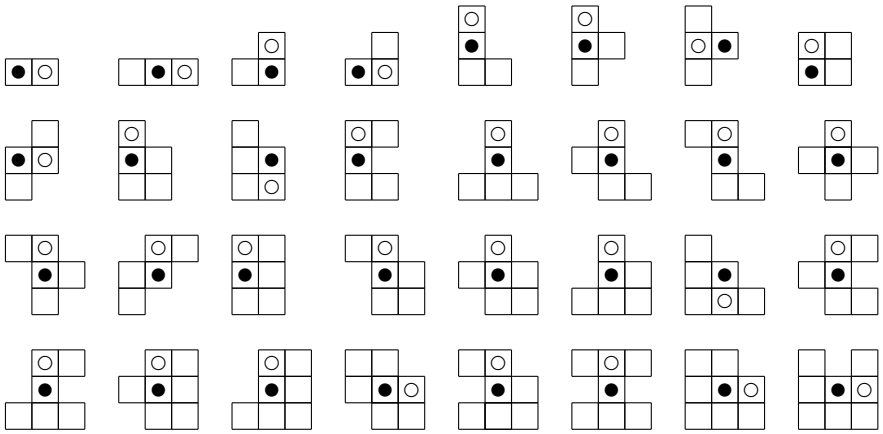


Fig. 2. When $t = 0$ the first movement of an agent located in (x, y) should be decided according to initial dirtiness status of the neighbors of (x, y), as appears in these charts — the agent's initial location is marked with a filled circle while the destination is marked with an empty one. All configurations that do not appear in these charts can be obtained by using rotations and mirroring.

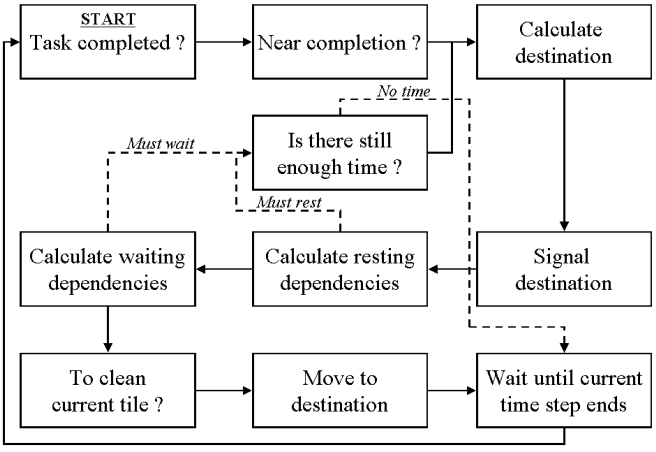


Fig. 3. A schematic flow chart of the **CLEAN** protocol. The smooth lines represent the basic flow of the protocol while the dashed lines represent cases in which the flow is interrupted. Such interruptions occur when an agent calculates that it must not move until other agents do so (either as a result of *waiting* or *resting* dependencies — see sections 5 and 6 of **CLEAN** for more details).

- **System Initialization** — initializing the system at the beginning of the agents operation.
  1. Arbitrarily choose the initial *pivot point* p_0 from all the points of ∂F_0.
  2. Artificially mark p_0 as a *critical point*.
  3. Place all the agents on p_0.
  4. For $(i = 1; i \leq k; i++)$ do
     (a) Call **Agent Reset** for agent i.
     (b) Call **CLEAN** for agent i.
     (c) Wait two time steps.
  5. End **System Initialization**.
- **Agent Reset** — resetting counters and flags.
  1. *resting* $\leftarrow$ *false*.
  2. *destination* $\leftarrow$ null.
  3. *cleaned in last tour* $\leftarrow$ *false*.
  4. *waiting* $\leftarrow \emptyset$.
  5. End **Agent Reset**.
- **Priority**.
  1. Assume the agent moved from tile (x_0, y_0) to tile (x_1, y_1) then
     (a) *priority* $\leftarrow 2(x_1 - x0) + (y_1 - y0)$.
  2. End **Priority**.
- **Check Completion of Mission**.
  1. If $((x, y) = p_0)$ and (x, y) has no dirty neighbors then
     (a) If (x, y) is dirty then
        i. Clean (x, y).
     (b) **STOP**.
  2. End **Check Completion of Mission**.
- **Check "Near Completion" of Mission** — identify scenarios in which there are too many agents, which are blocking each other, preventing the cleaning of the last few dirty tiles.
  1. If (this is the first time this function is called for this agent) then
     (a) Jump to 3.
  2. If $((x, y) = p_0)$ and (this is the end of a tour) then
     (a) If (*cleaned in last tour* = *false*) then
        i. *resting* $\leftarrow$ *true*.
        ii. **STOP**.
     (b) Else
        i. *cleaned in last tour* $\leftarrow$ *false*.
  3. End **Check "Near Completion" of Mission**.

Fig. 4. The first part of the **CLEAN** cleaning protocol. The terms ∂F and $\tau_i = (x, y)$ are defined in sections 4.1 and 4.2.

– **CLEAN Protocol** — controls agent number i actions after **Agent Reset** and until the mission is completed.
  1. **Check Completion of Mission**.
  2. **Check "Near Completion" of Mission**.
  3. Calculate destination of movement.
     (a) *destination* $\leftarrow$ *rightmost neighbor* of (x, y).
  4. Signaling the desired destination.
     (a) *destination signal bits* $\leftarrow$ *destination*.
  5. Calculate resting dependencies — solves the agents clustering problem.
     (a) Let all agents in (x, y) except agent i be divided into the following four groups :
        - A_1 : Agents signaling towards any direction different than *destination*.
        - A_2 : Agents signaling towards *destination* which entered (x, y) before agent i.
        - A_3 : Agents signaling towards *destination* which entered (x, y) after agent i.
        - A_4 : Agents signaling towards *destination* which entered (x, y) in the same time step as agent i.

     (b) Let group A_4 be divided into the following two groups :
        - A_{4a} : Agents with lower *priority* than this of agent i.
        - A_{4b} : Agents with higher *priority* than this of agent i.

     (c) *resting* $\leftarrow$ *false*.
     (d) If ($A_2 \neq \emptyset$) or ($A_{4b} \neq \emptyset$) then
        i. *resting* $\leftarrow$ *true*.
        ii. If (current time step did not end yet) then jump to 3 else jump to 9.
  6. Calculate waiting dependencies — takes care of agents synchronization.
     (a) *waiting* $\leftarrow \emptyset$.
     (b) If $(x-1, y) \in F_t$ and contains a *non-resting* agent which didn't move in the current time step yet then *waiting* $\leftarrow$ *waiting* $\cup$ {*left*}.
     (c) If $(x, y-1) \in F_t$ and contains a *non-resting* agent which didn't move in the current time step yet then *waiting* $\leftarrow$ *waiting* $\cup$ {*down*}.
     (d) If $(x-1, y-1) \in F_t$ and contains a *non-resting* agent which didn't move in the current time step yet then *waiting* $\leftarrow$ *waiting* $\cup$ {*left-down*}.
     (e) If $(x+1, y-1) \in F_t$ and contains a *non-resting* agent which didn't move in the current time step yet then *waiting* $\leftarrow$ *waiting* $\cup$ {*right-down*}.
     (f) If *destination* = *right* and tile $(x+1, y)$ contains an agent j, and *destination*$_j \neq$ *left*, and there are no other agents delayed by the operating agent (i.e. tile $(x-1, y)$ does not contain an agent l where *destination*$_l$ =*right* and tile $(x, y+1)$ does not contain any agents and tile $(x+1, y)$ does not contain an agent n where *destination*$_n$ = *left*), then (*waiting* $\leftarrow$ *waiting* $\cup$ {*right*}) and (*waiting*$_j \leftarrow$ *waiting*$_j \setminus$ {*left*}).
     (g) If *destination* = *up* and tile $(x, y+1)$ contains an agent j, and *destination*$_j \neq$ *down*, and there are no other agents delayed by the operating agent (i.e. tile $(x, y-1)$ does not contain an agent l where *destination*$_l$ =*up* and tile $(x+1, y)$ does not contain any agents and tile $(x, y+1)$ does not contain an agent n where *destination*$_n$ = *down*), then (*waiting* $\leftarrow$ *waiting* $\cup$ {*up*}) and (*waiting*$_j \leftarrow$ *waiting*$_j \setminus$ {*down*}).
     (h) If (*waiting* $\neq \emptyset$) then
        i. If (current time step did not end yet) then jump to 3 else jump to 9.
  7. Decide whether to clean (x, y) (meaning — whether or not the tile is a *critical point*).
     (a) If (x, y) has two dirty tiles in its $4Neighbors$ which are not connected via a sequence of dirty tiles from its $8Neighbors$ then
        i. (x, y) is a *critical point* and should not be cleaned.
     (b) Else
        i. If (x, y) still has other agents in it then
           A. Do not be cleaned (x, y).
        ii. Else
           A. Clean (x, y).
           B. *cleaned in last tour* $\leftarrow$ *true*.
  8. Move to *destination*.
  9. Wait until the current time step ends.
  10. Return to 1.

Fig. 5. The **CLEAN** cleaning protocol. The term *rightmost neighbor* is defined in section 4.2.

4.4 Signaling

Since each agent's sensors can detect tiles the status of all tiles which are contained within a digital sphere of radius four placed around the current location of the agent, each agent can artificially calculate the desired destination of all the agents which are located in one of its $4Neighbors$ tiles. Thus, the *signaling* action of each agent can be simulated by the other agents near him, and hence an explicit signaling by the agents is not actually required. However, the signaling action is kept in the description and flowchart of the protocol for the sakes of simplicity and understandability.

4.5 Connectivity Preservation

The connectivity of the region yet to be cleaned, F, is preserved by allowing an agent to clean only *non-critical* points. This guarantees both successful termination and agreement of completion (since having no dirty neighbors implies that $F = \emptyset$). Also, should several agents malfunction and halt, as long as there are still functioning agents, the mission will still be carried out, albeit slower.

Note that a tile which was originally clean, or which was cleaned by the agents, is guaranteed to remain clean. As a tile can be cleaned by the agents only when it is in ∂F, it is easy to see that the simple connectivity of F is maintained throughout the cleaning process (as the creation of "holes" — clean tiles which are surrounded by tiles of F — is impossible).

Note that when several agents are located in the same tile, only the last one who exits cleans it (see section 7(b)iA in **CLEAN**), in order to prevent agents from being 'thrown' out of the dirty region (meaning, cleaning a tile in which another agent is located and thus preventing this agent from being able to continue the execution of the cleaning protocol). An alternative method for ensuring the agents are always capable of executing the cleaning protocol would have been the implementation of a "dirtiness seeking mechanism" (i.e. by applying methods such as suggested in [Bae1]). Such a mechanism would have allowed an agent to clean its current location, even if other agents had also been present there. That solution, however, would have been far less elegant and would have added additional difficulties to the analysis process.

4.6 Agents Synchronization

Note that agents operating in the described environment must have some mean of synchronization, which is mandatory in order to prevent agents from operating in the same time, which may damage the dirty region's connectivity, as shown in Figure 6.

To ensure such scenarios will not occur, an order between the operating agents must be implemented. Note — throughout the next paragraphs agents who are signalling a *resting* status (see Section 4.7 for more details) are disregarded while calculating the dependencies of the agents' movements. The following suggested order is implemented in section 6 of **CLEAN** :

For agent i, let $P_i \subseteq \{up, down, left, right\}$ be a set of directions of tiles, in which there are currently agents, which agent i is delayed by (meaning, agent i will not start moving until the agents in these tiles move). Unless stated otherwise, $P_i = \emptyset$.

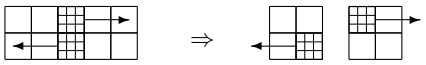


Fig. 6. When the agents do not possess a synchronization mechanism, they may, among others, damage the region's connectivity. In this example, two agents clean their current locations, and move according to the **CLEAN** protocol. Since they are not synchronized, the tiles which they are located in are not treated as critical at the time of cleaning, but the region's connectivity is not preserved. Should one agent had waited for the other agent to complete its actions, it would not have cleaned its current tile, and the region's connectivity would have been maintained.

For agent i which is located in tile (x, y), if $(x-1, y)$ is a tile in F, which contains an agent, then $P_i \leftarrow P_i \cup \{left\}$. if $(x, y-1)$ is a tile in F, which contains an agent, then $P_i \leftarrow P_i \cup \{down\}$.

Let $dest_i \in \{up, down, left, right\}$ be the destination agent i is interested in moving to, after cleaning its current location.

Let each agent be equipped with a two bits flag (implemented by two small lightbulbs, for example). This flag will state the desired destination of the agent. Alternatively, each tile can be treated as a physical tile, in which the agent can move. Thus, the agent can move towards the top side of the tile, which will be equivalent for using the flag in order to signal that it intends to move up.

Let each time step be divided into two phases. In phase 1, every agent "signals" the destination it intends to move towards, either by moving to the appropriate side of the tile, or by using the destination flag.

As we defined an artificial rule which states the superiority of $left$ and $down$ over $right$ and up, there are several specific scenarios in which this asymmetry should be reversed in order to ensure a proper operation of the agents. Following is such a a "dependencies switching" rule: For agent i, which is located in (x, y), if $dest_i = right$ and tile $(x+1, y)$ contains an agent j, and $dest_j \neq left$, and there are no other agents which are delayed by agent i (i.e. tile $(x-1, y)$ does not contain an agent l where $dest_l = right$ and tile $(x, y+1)$ does not contain any agent and tile $(x+1, y)$ does not contain an agent n where $dest_n = left$), then $P_i \leftarrow P_i \cup \{right\}$ and $P_i \leftarrow P_j \setminus \{left\}$. Also, if $dest_i = up$ and tile $(x, y+1)$ contains an agent k, and $dest_k \neq down$, and there are no other agents which are delayed by agent i (i.e. tile $(x, y-1)$ does not contain an agent m where $dest_m = up$ and tile $(x+1, y)$ does not contain any agent and tile $(x, y+1)$ does not contain an agent q where $dest_q = down$), then $P_i \leftarrow P_i \cup \{up\}$ and $P_i \leftarrow P_k \setminus \{down\}$.

At phase 2 of the time step, the agents start to operate in turns, according to order implied by P_i. This guarantees that the connectivity of the region is kept, as a simultaneous movement of two neighboring agents is prevented.

Notice that deadlocks are impossible — since the basic rule is that every agent is delayed by the agents in its $left$ and $down$ neighbor tile, at any given time among the agents who had not moved yet in the current time step, there is always the agent with the minimal x and y coordinates, which is able to move. After this agent moves, all the agents which are delayed by it, can now move, and so on. As to the "dependencies switching rule" — let agent i located in tile (x, y) have the minimal x and y values

among the agents who had not moved yet, and let $dest_i = up$, and let tile $(x, y+1)$ contain an agent j such that $dest_j \neq down$. Then although agent i is located below agent j, it will be delayed by it (i.e. $(up \in P_i)$ and $\neq (down \in P_j)$) as long as agent i is not delaying any other agents (as this is the requirement of the "dependencies switching rule"). In this case, we should show that there can not be a circle of dependencies, which starts at agent j, ends at agent i, and is closed by the dependency of agent i on agent j. Such a circle can not exist since for it to end in agent i, it means that agent i is delaying another agent k. However, this is impossible since agent i not delaying any other agents (specifically agent k) was a strict requirement of the "dependencies switching rule" which allowed agent i to delay agent j. Thus, a dependencies circle is prevented and no deadlock are possible.

Note that while phase 2 is in process, F_t may change due to the cleaning of the agents. As a result, the desired destinations of the agents as well as their dependencies forest, must also be dynamic. This is achieved through the constant recalculation of these values, for every $waiting$ agent. For example, assume agent i to be located in (x, y), and $dest_i = down$ without loss of generality, and let agent j located in tile $(x, y-1)$ moves out of this tile and cleans it. Then, $dest_i$ naturally change (as the tile (x, y) does no longer belong to F_t, and thus is not a legitimate destination for agent i), and thus, P_i may also change. In this case, agent i should change its "destination signal" and act according to its new $dest_i$ and P_i. This is implemented in **CLEAN** by calculating the waiting agents' destinations and dependencies lists repeatedly, until either all agents have moved or until the time step has ended (meaning that some agents had to change their status to $resting$, and pause until the next time step — see section 4.7 for more details). Note that every $waiting$ agent is guaranteed either to complete its movement in the current time step, or to be forced to wait for the next time step, by switching its status to $resting$.

Notice that the "dependencies switching rule" is not required in order to ensure a proper completion of the mission, but rather to improve the agents' performance, by preventing a bug demonstrated in Figure 7.

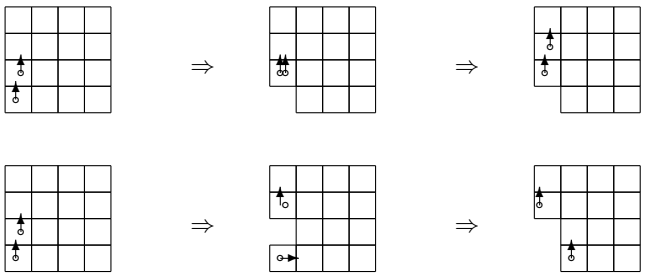


Fig. 7. The upper charts demonstrate a performance bug which may be caused due to the local dependencies. The agents are advancing according to the CLEAN protocol, but their cleaning performance is decreased. The lower charts demonstrates the cleaning operation after adding the dependencies switching mechanism.

4.7 Clustering Problem

Since we are interested in preventing the agents from moving together and functioning as a single agent (due to a resulting decrease in the system's performance), we would like to impose a "resting policy" which will ensure that the agents do not group into clusters which move together. This is done by artificially delaying agents in their current locations, should several agents placed in the same tile wish to move to the same destination. However, we would like the resting time of the agents to be minimal, for performance and analysis reasons.

The following resting policy is implemented by section 5 of **CLEAN**. Using its sensors, an agent intending to move into tile v is aware of other agents which intend to move into v as well. Thus, when an agent enters a tile which already contains other agents, it knows whether these agents entered this tile at the same time step it did, or whether they had occupied this tile before the current time step.

Note that in phase 1 of each time step all the agents are signaling their desired destinations. Thus, an agent can identify which ones of the agents which are located in its current tile intend to move to the same destination as its. Only those agents may cause this agent to delay its actions and rest.

From the agents which intend to move to the same direction as agent i, the agent can distinguish between three groups of agents — the agents which entered this tile before the time step agent i did (group A_2), those which entered the tile in the same time step as i (group A_4), and those who entered it after agent i did (group A_3). If $A_2 \neq \emptyset$, agent i waits, change its status to *resting agent*, signals this status to the other agents in its vicinity, and does not move. As this rule is kept by every other agent, agent i is guaranteed that the agents of group A_3 in their turn will wait for agent i moves before being free to do so as well.

Regarding the agents in A_4, the problem is solved by induction over the time steps, using the assumption that two agents can not leave the same tile towards the same direction in the same time step. This of course holds for the first time steps, as the agents are released manually by the **CLEAN** protocol's initialization procedure. Later, as we are assured that all the agents of group A_4 arrived to v from different tiles (which are also different than the tile agent i had entered v from), then since all the agents in group A_4 know the previous locations of each other, a consensus over a local ordering of A_4 is established (note that no explicit communication is needed to form this ordering). An example for such an order is the **priority** function of the **CLEAN** protocol. As a result, the agents are able to exit the tile they are currently located in in an orderly fashion, according to a well defined order. Thus, the following invariant holds : "*at any given time t, for any two tiles v, u, there can only be a single agent which moves from v to u at time step t*". Thus, the clustering problem is solved.

4.8 Mission Termination

When $k > 1$ there may evolve scenarios in which k will be greater then $|F_t|$, the number of dirty tiles. In such a case, the agents may also be arranged in such a way that at any given time there is at least one agent in each tile. From such a scenario, agents may switch places without being able to clean a single tile, since only an agent which is

the last to exit a tile may clean it. The result of this situation would have been a group of agents running around infinitely without ever completing their cleaning mission. In order to prevent this scenario, the **CLEAN** protocol instructs any agent who reaches the pivot point and had not cleaned even a single tile in its last traversal of F, to halt (see *near completion of mission* procedure of **CLEAN**). When an agent did not clean any tile in its last traversal of the shape, it means that either the shape is a "skeleton" comprising of only critical points (see the proof of Theorem 2 in section 5.3) or that $k' > |F_t|$ (k' denoting the currently active agents). In either case, halting this agent does not slow the completion of the cleaning task. It is easy to see that if the number of active agents outside p_0 is always kept smaller than $|F_t|$ then the size of the shape will always be decreased until the shape is entirely cleaned.

5 Analysis

The introduction of the notion of critical points makes time-complexity analysis of the **CLEAN** protocol significantly harder since a critical point may be visited several times before its cleaning. We conjecture that the algorithm proposed is efficient, and additional agents will speed up the cleaning process only up to a limit. In order to give this argument a rigorous form, some definitions are required.

5.1 Definitions

Let s_t denote the size of the dirty region F at time t, namely the number of grid points (or tiles) in F_t. Actually, F defines a dichotomy of $\mathbf{Z}^2$ into F and $\overline{F} = \mathbf{Z}^2 \setminus F$.

The boundary of the dirty region F is denoted as ∂F and defined as :

$$\partial F = \{(x,y) \mid (x,y) \in F \ \wedge \ (x,y) \text{ has an } 8Neighbor \text{ in } (G \setminus F)\}$$

(as also defined in a previous section).

A *path* in F is defined to be a sequence $(v_0, v_1, \ldots, v_n)$ of tiles in F such that every two consecutive tiles are 4 $connected$ (the *Manhattan* distance between them is 1). The *length* of a *path* is defined to be the number of tiles in it.

Let tile v be called a *critical point* if there exist $v_1, v_2 \in 4Neighbors(v)$ for which all paths connecting v_1 and v_2, included in $8Neighbors(v)$, necessarily pass through v (where v, v_1, v_2 and all said paths are from F).

We shall denote by c_t the circumference of F at time t, defined as follows: let v_0 and v_n be two 4 $connected$ tiles in ∂F_t, and let $C_t = (v_0, v_1, \ldots, v_n)$ be the shortest *path* connecting v_0 and v_n, which contains all the tiles of ∂F_t and only such tiles. If there are several different shortest paths then let C_t be a one of them. Notice that C_t may contain several instances of the same tile, if this tile is a *critical point*(meaning that C_t is an ordering of the tiles of ∂F_t, in which multiple instances of tiles that are *critical points* are allowed). c_t will be defined as the length of C_t. An example appears in Figure 8.

For some $v \in F_t$ let $Strings_t(v)$ denote the set of all *paths* in F_t that begin in v and end at any *non-critical point* in ∂F_t, and let $w(F_t, v)$ denote the *depth* of v — the

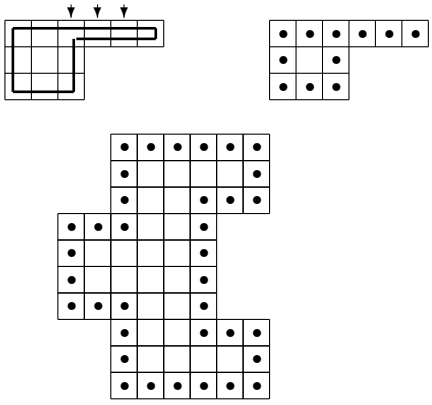


Fig. 8. The line in the left upper chart goes through the tiles of C_t where the 3 arrows denote tiles that are included twice. The circles in the right upper chart denote the tiles of ∂F_t. Note that while ∂F_t contains 11 tiles, C_t contains 14. In the lower chart, there are no *critical points* in ∂F_t and therefore $c_t = |\partial F_t| = |C_t| = 36$.

length of the shortest *path* in $Strings_t(v)$ (unless v is a *critical point* in which case its *depth* is defined to be zero).

Let $W(F_t)$ denote the *width* of F_t, defined as the maximal depth of all the tiles in F_t, i.e. :

$$W(F_t) = \max\{w(F_t, v) \mid v \in F_t\}$$

An example appears in Figure 9.

Let F'_t denote the region one could get, having one agent traverse F_t once, using the **CLEAN** protocol (i.e. when $k = 1$, $F'_t = F_{t+c_t}$). For the sake of simplicity, F''_t will be used instead of $(F'_t)'$, and so on.

The longest in-region distance between p_0 and any other point in F_t will be referred to as $l(F_t)$, the *length* of F_t:

$$l(F_t) = \max_{v \in F_t} \{d_{F_t}(p_0, v)\}$$

where $d_{F_t}(x, y)$ is the distance (shortest path within F_t) between x and y.

5.2 Correctness

Lemma 1. *If $s_t > 0$ then $W(F_t) \geq 1$, that is: any finite simple region has at least two non-critical points on its boundary.*

Proof. The boundary ∂F_t is a connected graph, hence it has a spanning tree. In such a tree there are at least two vertices with degree 1. these vertices are necessarily not critical in F_t, since any boundary point which is critical in F_t is also critical in ∂F_t.

(End of Lemma 1)

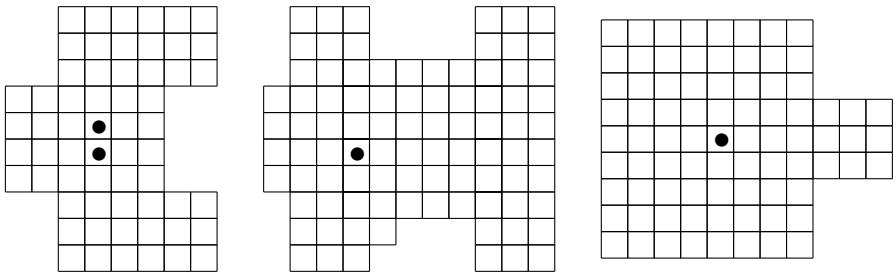


Fig. 9. The chart contains three shapes. The black circles denote the deepest tiles within the shapes. Notice that while in the left shape there are two tiles whose depth is the maximal, in the other shapes there is only one tile of maximal depth. The widths of the shapes equal the depth of the deepest tiles, and are (from left to right) 2, 3, and 4.

Lemma 2. *During the course of the cleaning process, if all the agents are using the **CLEAN** protocol the agents are never going outside the dirty region. Namely, the only time an agent is located on a clean tile is when $F = \emptyset$.*

Proof. The only movements allowed by the **CLEAN** protocol are towards a dirty tile. The only times an agent is allowed to clean a tile are when it is the last agent leaving this tile, or when this is the only dirty tile left. In addition, the various preemption which are in charge of agents synchronization prevent an agent to move to a dirty tile, which is getting cleaned during its movement. As a result, an agent is guaranteed to be located in dirty tiles throughout the entire cleaning process.

(End of Lemma 2)

Theorem 1. *A group of agents executing the **CLEAN** protocol will eventually clean a simply connected region and stop together at the pivot point p_0.*

Proof. While F_t has not yet been cleaned, $s_t > 0$ and hence, by Lemma 1 there is a non-critical point on ∂F_t. Since the agents obey the **CLEAN** protocol, while $F_t \neq \emptyset$, there is at least a single agent still traversing F_t. As this agent does not change the direction of its traversal, it is guaranteed to arrive to (and clean) a non-critical point at least once during the course of its traversal (thus decreasing s_t by at least 1). As $c_t \leq 4s_t$ (since during a traversal around F_t any tile can be entered to from each one of its 4 neighbors at most once), the maximal length of this traversal is $4s_t$. As agents may indeed delay one another, it is shown in Lemma 5 that the actual time required for an agent to complete its traversal around F_t is at most four times the length of this traversal, namely — $16s_t$. As $\forall t, s_t \leq s_0$, it can be seen that after no more than $16s_0^2$ time steps we shall have $s_t = 0$. The fact that all agents will meet at the same point is implied by the following two rules that are implemented in the **CLEAN** algorithm:

– **rule 1:** F_t is always being kept connected. (We never clean a tile that has no clean neighbor and never clean a critical tile either).

– **rule 2:** The pivot p_0 is cleaned only when no other dirty points are left in F_t.

(End of Theorem 1)

5.3 Detailed Analysis

In this section, we shall discuss an upper bound for the cleaning time of k agents employing the **CLEAN** cleaning protocol, for some dirty shape F_0. From an upper bound for the cleaning time of a swarm comprising k agents, an upper bound for the number of agents needed to ensure a successful cleaning of a given shape in a given number of time-steps can be derived.

The following Lemma states the change in the cardinality of a shape's circumference after being traversed by an agent using the **CLEAN** protocol.

Lemma 3. *The cardinality of the region's circumference always decreases after being traversed by an agent applying the **CLEAN** protocol, namely :*

$$|\partial F'| \leq |\partial F| - 8$$

Proof. Note that a traversal around F is a closed, simple, rectilinear polygon. $\partial F'$ was obtained after deleting all the *non-critical* points of ∂F. Traversing such a polygon, an agent either goes straight, makes an internal turn ("left" turn, if we assume a clockwise movement), or makes an external turn ("right" turn). Suppose w.l.o.g that an agent is moving up and making an internal turn left. Assume that there are no critical points. The path around this turn, along the newly created boundary tiles is now longer by two, as it contains an additional movement up, and another one towards the left. Similarly, an external turn, creates a new path which is shorter by two tile (as a single horizontal movement and a single vertical movement are no longer necessary). Since ∂F is a simple rectilinear polygon, it always has four "right" turns more than "left" ones[3]. When *critical* points are met, they are not being cleaned, which means that they exist in both ∂F and $\partial F'$. Re-visiting these points, however, does not change the overall size of the set of tiles (see an example in Figure 10).

(End of Lemma 3)

While producing a bound for the cleaning time of the agents we shall demonstrate that while being traversed by the agents using the **CLEAN** protocol, the width of the initial shape F_0 decreases monotonically. The main component of this proof is presented in the following Lemma.

Lemma 4. *Every time a region is traversed by an agent using the **CLEAN** protocol, its width is decreased by at least one, namely :*

$$W(F'_t) \leq W(F_t) - 1$$

[3] This is a simple consequence of the "rotation index" Theorem (see e.g. [DoC1] pp. 396) : If $\alpha : [0,1] \rightarrow R^2$ is a plane, regular, simple, closed curve, then $\int_0^1 k(s)ds = 2\pi$, where $k(s)$ is the curvature of $\alpha(s)$ and the curve is traversed in the positive direction.

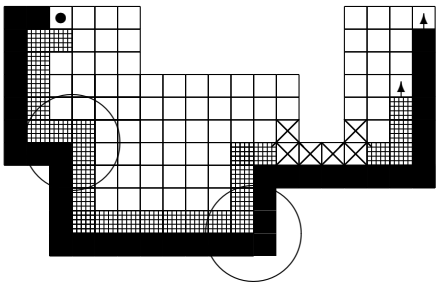


Fig. 10. An example of Lemma 3 and of the **CLEAN** protocol. The black dot denotes the starting point of the agents cleaning F (which is the entire shape presented). The X's mark the *critical points* which are not cleaned. The black tiles are some of the tile of ∂F, cleaned by the first agent. The second layer of marked tiles represent some of the tiles of $\partial F'$, cleaned by the second agent. The effect of corners on the traversal cardinality is either an increase (marked by the left circle) or a decrease (marked by the right circle).

A corollary is that the number of tours around a region F that k agents must accomplish before $F = \emptyset$ is at most $(\frac{W(F)}{k} + 1)$.

Proof. By the definition of the depth of a tile, it is the shortest path to a *non-critical* tile in ∂F. According to the **CLEAN** protocol, after an agent had traversed F, all of the *non-critical points* in ∂F were cleaned. This is true as the agent is instructed to clean all the *non-critical points* it is going through. The only exception are tiles which an agent does not clean upon exit, as there are other agents currently located in it. However, in this case, this tile will be cleaned by the last agent leaving it. The *resting* mechanism (discussed in Section 4.7) prevents situations in which a certain agent is never leaving the tile it is located in. Therefore, the only time in which a tile might constantly contain agents, is when the number of agents is greater than the number of tiles. The termination mechanism (discussed in Section 4.8) is in charge of detecting such scenarios and stopping the operation of agents, until the number of active agents decreases.

Therefore, after an agent traverses the region, the depth of every internal tile was decreased by (at least) one, meaning that the total width of the shape was decreased by (at least) one. As a result, as all agents are identical, when k agents complete a traversal of F, the width of F is decreased by at least k.

(End of Lemma 4)

When examining the cleaning time of the agents, one must remember that merely calculating the length of the paths the agents must move along is not enough. Since in various scenarios the **CLEAN** protocol may direct an agent to stop and wait (for example, when several agents are located in the same tile), a way to bound the actual time it takes an agent to move along a specific path must be found. The following Lemma establishes the relation between the length of a path and the maximal time it takes an agent using the **CLEAN** protocol to move along it.

Lemma 5. *The time it takes an agent which uses the **CLEAN** protocol to move along a path of length c_t (including delays caused by other agents located in the same tiles) is at most $4 \cdot c_t$.*

Proof. According to the problem's specifications, each agent moves along the dirty region F at a pace of one tile per time step. The only exception may occur when several agents are delayed after entering the same tile.

According to the **CLEAN** protocol, although several agents can enter the same tile at the same time step, agents located in the same tile can leave it at the same time step only if they do so towards different directions. As a result, there arises the question whether more and more agents can enter a certain tile without leaving it, causing a decrease in the swarm's performance.

Let v be an empty tile. Let us assume w.l.o.g that at some time step t, 4 agents enter v (this is of course the maximal number of agents which can enter v at the same time step, due to the invariant stating that no two agents can exit u to v at the same time step, for u, some neighbor of v, and since v has at most 4 neighbors). Thus, in time step $t-1$ v had exactly 4 neighbors in ∂F. Let us assume that in time step $t+1$ v still has 4 neighbors in ∂F. Thus, all the 4 agents which entered v intends to move to 4 different directions (since according to the **CLEAN** protocol, each of them has different *rightmost* neighbor). Thus, none of them will be delayed. Alternatively, let us assume that at time step $t+1$ v has less than 4 neighbors in ∂F. Let α denote the number of neighbors of v in ∂F. Then, since the 4 agents which entered v in time t did so from different directions, according to the **CLEAN** protocol, α of them will be able to leave v towards the α neighbors of v, and $(4-\alpha)$ will stay in v. However, in this time step, only α new agents can enter v, since v has only α neighbors in ∂F. Thus, in time step $t+2$ there are at most 4 agents in v. This is true of course for each time steps $T > t+2$. Since the number of agents in each tile is at most 4, and since each tile with agents in it has at least one neighbor of ∂F (since the agents had to enter it from a neighbor in ∂F, then there are at least $\frac{k}{4}$ agents which are able to move. Thus, even if the agents collide with each other continuously, the time it takes k agents to traverse a region of circumference c_t is at most $4 \cdot k \cdot c_t$ (and the average traversal time of an agent, amortized for the entire group of agents, is at most 4 times its minimal traversal time). In reality, the agents' traversal time is only slightly more than c_t, although an analytic proof is yet to be found).

(End of Lemma 5)

When an agent using the **CLEAN** protocol is traversing F_t it does so by moving along a path which contains all the tiles of ∂F_t. Since by doing that, the agent may pass in the same tile more than once, the length of this path may be larger than the number of tiles in ∂F_t. A bound for the ratio between the two is described in the following Lemma.

Lemma 6. *c_t, the length of the circumference of F_t never exceeds twice its cardinality, namely :*

$$c_t \leq 2 \cdot |\partial F_t| - 2$$

Proof. ∂F_t is a connected graph and thus it has spanning trees. A protocol for constructing a spanning tree for ∂F_t and a *Depth First Search* (DFS) scan of it was constructed,

such that the path generated by using the DFS algorithm contains a traversal around F_t (meaning that the DFS scan generates a path which after having several of its tiles removed, equals a path which traverses F_t). An example appears in Figure 12.

The protocol receives p_s, a *non-critical* tile as its starting point and an empty list of tiles, L.[4] The protocol constructs a spanning tree of ∂F_t as well as a DFS scan for this tree, by adding tiles to the list. The protocol also marks the tiles in L which should be deleted. After the deletion of these tiles the remaining tiles form a path which traverses F_t. An example of the above appears in Figure 12.

A DFS scan of a tree can go through each edge at most twice. A tree of $|V|$ vertices contains exactly $(|V|-1)$ edges. Thus, a DFS scan of a spanning tree of ∂F_t contains at most $2 \cdot (|\partial F_t| - 1)$ transitions of edges. Since there exists such a spanning tree that contains a 'tour' around F_t then : $c_t \leq 2 \cdot |\partial F_t| - 2$.

(End of Lemma 6)

Protocol **SPAN-DFS**(x, y) :
Add (x, y) to L and mark it as **OLD**;
If the *rightmost* neighbor of (x, y) is p_s then
    Delete all the marked tiles from L and **STOP**;
If (the *rightmost* neighbor of (x, y) is marked as **OLD**) and (no circle was formed) then
    / The traversal repeats this tile as well */*
    Call **SPAN-DFS** for the *rightmost* neighbor of (x, y) and **STOP** upon return;
If (the *rightmost* neighbor of (x, y) is marked as **OLD**) and (a circle was formed) then
    / Continue, and clean the redundant tiles */*
    Add to L the sequence of tiles from L, starting with (x, y) and backwards to (x, y)'s *rightmost* neighbor;
    Mark these tiles (expect (x, y)) to be deleted;
    Call **SPAN-DFS** for the *rightmost* neighbor of (x, y) and **STOP** upon return;
/ The rightmost neighbor was not OLD */*
Call **SPAN-DFS** for the *rightmost* neighbor of (x, y) and **STOP** upon return;
End **SPAN-DFS**;

Fig. 11. The recursive spanning tree construction protocol. The term *rightmost* has the same meaning as in the **CLEAN** protocol. By checking L we can find out whether continuing to an **OLD** neighbor is a "clean return" (e.g. (A,B,C,B)) or whether it completes a circle (e.g. (A,B,C,A)).

[4] The existence of a *non-critical* point is guaranteed since ∂F_t is a connected graph and thus has a spanning tree, in which at least two tiles has a degree of 1, which makes them *non-critical* tiles.

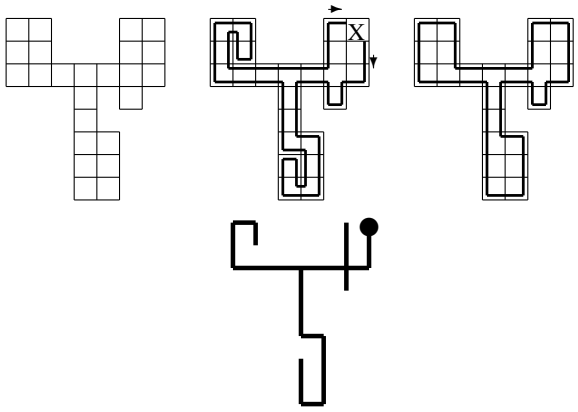


Fig. 12. An example of the spanning tree protocol. The left chart represents the dirty region F_t. The middle chart shows the DFS scan of ∂F_t according to the **SPAN-DFS** protocol, when the big X marks p_s, the *non-critical* starting point. The right chart shows the traversal path which is contained in this DFS scan. The drawing on the bottom shows the spanning tree that is created by the protocol and is searched by the DFS algorithm.

Let $W_{REMOVED}(t)$ denote the decrease in the width of F due to the agents' activities until time t.

Lemma 7.

$$W_{REMOVED}(t+1) \geq \left\lfloor \sum_{i=1}^{t} \frac{k}{4 \cdot c_i} \right\rfloor$$

Proof. $W_{REMOVED}(t)$ can be defined as the number of completed traversals around the dirty region, multiplied by the number of the agents k. Note that at time step t, each agent completes $\frac{1}{c_t}$ of the circumference. Since we know the value of c_t for every t, and since we know that the time it takes an agent to move along a path is at most 4 times the length of this path, then the decrease in the original width of the shapes caused by the cleaning process of the agents is bounded as described.

(End of Lemma 7)

A bound over the cleaning time of a given shape F_0 can then be produced as follows :

Theorem 2. *Assume that k agents start cleaning a simple connected region F_0 at some boundary point p_0 and work according to the **CLEAN** algorithm, and denote by $t_{success}(k)$ the time needed for this group to clean F_0. Then it holds that:*

$$\max\left\{2k, \left\lceil \frac{s_0}{k} \right\rceil, 2l(F_0)\right\} \leq t_{success}(k) \leq \frac{8(|\partial F_0| - 1) \cdot (W(F_0) + k)}{k} + 2k$$

Proof. The lower bound is quite obvious — the left term $2k$ is the time needed to release the k agents from the pivot point, $\frac{s_0}{k}$ is a lower bound on the time necessary to cover

the region if the agents were optimally located at the beginning. The right term, $2l(F_0)$, comes from the observation that at least one agent should visit (and return from) any point of F_0, including the one farthest from p_0, to which the distance is $l(F_0)$.

Considering the upper bound, in order for F_0 to be cleaned, there should exist a time $t_{success}$ in which the width of the shape will be 0. Note that once the dirty region was reduced to a 'skeleton' comprises only of tiles of depth zero, an additional traversal must be done in order to clean it entirely.

By combining Lemma 3 and 6 we know that for $c_j(i)$ and $\partial F_j(i)$ be the length and cardinality respectively of the i-th traversal of agent j :

$$c_j(i) \leq 2(|\partial F_j(i)| - 1) \leq 2(|\partial F_0| - 1)$$

Using Lemma 7 we see that :

$$W_{REMOVED}(t) \geq \left\lfloor \sum_{i=1}^{t} \frac{k}{4 \cdot c_i} \right\rfloor \geq \left\lfloor \frac{k \cdot t}{8(|\partial F_0| - 1)} \right\rfloor \tag{1}$$

Thus, we are interested in :

$$\left\lfloor \frac{k \cdot t_{success}(k)}{8(|\partial F_0| - 1)} \right\rfloor \geq W(F_0) + k \tag{2}$$

adding the "$+k$" for the additional traversal needed for cleaning the dirty 'skeleton'.

Since releasing the agents requires an additional $2k$ time steps, the final bound for this case is :

$$t_{success}(k) \geq \frac{8(|\partial F_0| - 1) \cdot (W(F_0) + k)}{k} + 2k$$

(End of Theorem 2)

Using the above theorem we can bound the *efficiency ratio*, defined as $\frac{t_{success}(k)}{s_0}$ which expresses the benefit of using k agents for a cleaning mission :

Corollary 1.

$$\max\left\{\frac{2k}{s_0}, \frac{1}{k}, \frac{2l(F_0)}{s_0}\right\} \leq \frac{t_{success}(k)}{s_0} \leq \frac{8\left(|\partial F_0| - 1\right) \cdot (W(F_0) + k)}{k \cdot s_0} + \frac{2k}{s_0} \tag{3}$$

An interesting result of Corollary 1 is that when $W(F_0) \ll k \ll s_0$, i.e. the number of agents is large relative to the width but small compared to the area, then the efficiency ratio is bounded below by twice the ratio of the length and area, and bounded above by $8\frac{|\partial F_0|}{s_0}$. Note here the similarity to the ratio $\frac{c_0}{\sqrt{s_0}}$, known as the *shape-factor*.

Another conclusion is that when we scale up the region by a factor of n, the area increases as n^2 but the width, length and circumference all increase as n so we get the following:

Corollary 2.

$$\textit{as } n \to \infty, \qquad L_\infty \leq \frac{t_{success}(k)}{S} \leq U_\infty$$

where

$$L_\infty = \frac{1}{k}, \qquad U_\infty = 8\frac{\partial F \cdot W}{k \cdot S} = 8\frac{|\partial F_0| \cdot W(F_0)}{k \cdot s_0}$$

and $S = s_0 n^2, \partial F = |\partial F_0| n, W = W(F_0) n$ *are the scaled area, circumference cardinality and width, respectively.*

5.4 Robustness

The issue of an algorithm's robustness and fault-tolerance is a major aspect of robotics systems, as the environments in which the they operate may often include unpredicted changed from the original specification of the system, such as various kinds of noises, robots' malfunctions, etc'.

As discussed in Section 4.1, one of the requirements of the CLEAN protocol is a complete fault-tolerance to malfunctions in the agents, meaning — even if one or several agents stop working ("die") the rest of the agents will continue the cleaning process as efficiently as their number allows them, and as long as there exist at least one agent who functions properly, the cleaning completion is guaranteed. The fulfillment of this requirement is immediately derived from the completeness of the algorithm for a group of agents at any size (and specifically, for a single agent), shown in Theorem 1. The basic idea enabling this robustness is the complete independence between the agents, due to the full decentralized nature of the system (as the agents do not share the cleaning mission between them, nor do they rely on one another in the performance of their cleaning process).

A different aspect of the algorithm's robustness is the performance of the agents in noisy environments. In general, noise can take place in various aspects of the system — while sensing whether a tile should be cleaned, while sensing for presence of other agents in an agent's vicinity, noises in the agent's movement system, etc'. As in almost any system, it is easy to see that an exaggerated noise level may significantly decrease the algorithm's efficiency, and may even prevent the agents from completion the mission altogether. For example, difficulties in sensing whether a tile should be cleaned or not, may cause an agent to clean a critical point, thus separating the region into several connected components. In the event that one of these components do not have any agent located on its tiles, it will remain dirty, even after the agents stop and report a successful termination of their mission. In addition, noises in the agents' movement system may cause the agents to detach from the dirty region, thus delaying, or even preventing the completion of the cleaning process. Whereas a situation in which the noise level is kept at a reasonable rate (i.e. correct identification of clean and dirty tiles, and movement noises), it is the authors' estimation that the completion of the algorithm can still be guaranteed, although the cleaning time may be longer. This may happen when the agent suffers difficulties in the sensors in charge of detecting other agents in its vicinity, and interpreting their signaling (see relevant sections for more details about signaling and synchronization). When this happens, several problematic scenarios, which these mechanisms were designed to avoid, may occur. However, simulations show that in such cases, only the cleaning time is affected, and not the completion of the cleaning itself. Nevertheless, as this issue was not fully investigated yet, a zero noise level is still one of the algorithm's requirements.

One example for the above is described in Figure 6, in which a failure in the sensing mechanism may damage the simple-connectivity of the dirty region. Note that in such scenarios, there is least a single agent in each of the new dirty components. Therefore, each component is an instance of the cooperative cleaners problem, whose complete cleaning is guaranteed. However, the completion of the cleaning for all of those components may require longer time than would have required for the original problem.

Another example is demonstrated in Figure 7, where a bug in the *waiting* mechanism (see Section 4.6 for more details) may prevent some of the agents from cleaning dirty tiles. In this example as well, the existence of at least a single fully functioning agent is guaranteed, and as a result, also the successfully completion of the cleaning mission. The same holds for possible bugs in the *resting* mechanism (see Section 4.7 for more details) which may cause several agents to move together, acting as a single cleaning agent.

It is also interesting to examine the scenario in which all the robots are picked up, and re-placed on some arbitrary positions onboard the dirty region. Thanks to the definition of the CLEAN protocol, the robots will navigate to the boundary of the region, and resume their cleaning. Naturally, the overall cleaning time might be affected, however — the completion of the cleaning is still guaranteed.

5.5 Comparison to Existing Work

Looking at the existing work that has been done with respect to multi robotic systems designed for a cooperative cleaning / coverage / etc', several analytic results concerning the completion time of the mission can be found.

An interesting result is presented in [Svn1]. In this work, a group of simple robots is used for periodically covering an unknown area, using the nodes counting method. Based on a more general result for undirected domains shown in [Koe3], the following bound is given :

> The cover time of teams of ant robots (of a given size) that use node counting on strongly connected undirected graphs can be exponential in the square root of the number of vertices.

Namely :

$$f(k) = O(2^{\sqrt{S_0}}) \tag{4}$$

denoting the covering time of k robots by $f(k)$.

Recalling Theorem 2 in which :

$$t_{success}(k) \leq \frac{8(|\partial F_0| - 1) \cdot (W(F_0) + k)}{k} + 2k$$

as $|\partial F_0| = O(S_0)$ and as $W(F_0) = O(\sqrt{S_0})$ we see that :

$$t_{success}(k) = O\left(\frac{1}{k} S_0^{1.5} + S_0 + k\right)$$

As for practical reasons we assume that $k < S_0$ we can see that :

$$t_{success}(k) = O\left(\frac{1}{k}S_0^{1.5} + S_0\right)$$

and when the number of robots is independent in the size of the region, the time complexity can be reduced to :

$$t_{success}(k) = O\left(S_0^{1.5}\right)$$

Comparing this to the bound of equation 4, we see that the time complexity of the **CLEAN** protocol is much smaller than this of the nodes counting technique.

It should be mentioned though, that in [Svn1] the authors clearly state that it is their belief that the coverage time of robots using nodes counting in grids is much smaller. This estimation is also demonstrated experimentally. However, it should also be pointed out that the methods described in [Svn1] require the robots to have some sort of *marking* capability (namely, leaving trails along the region's tiles). Such a requirement is not necessary for the **CLEAN** protocol.

6 Regions With Obstacles

So far we have only dealt with simply connected regions, i.e. — regions with no "holes". In the case of a (connected) region with obstacles (i.e. holes) the simple **CLEAN**ing algorithm will not work, due to the following "cycle" problem : eventually, each obstacle will be surrounded by critical points, and there will be a time when *all* boundary points of F will be critical (we shall call such a situation *useless*, as opposed to the *useful* state when some points are cleaned during a tour). As a cure to this problem, we suggest to add an "odoring" feature to our cleaners, that is — an agent will be able to label each tile it is going through by a small amount of "perfume" (this action may remind one of the pheromones left by ants during their walks). These labels will designate the directions according to which agents traveled through the labeled tiles. Upon getting to the useless state (detected by each agent due to no cleaning between two consecutive visits from the same entrance to the pivot point) an agent will continue to traverse the boundary, but will now look for a point which has a single label — that is, one that has odor pointing only on one direction of movement. If this tile does not have a neighbor which is entirely unlabeled, the agent will clean this point (despite its "criticality", as it is not really critical since it is necessarily part of a cycle around an obstacle) and then continue as in a useful state (see Figure 13 for an example). This will open one cycle, hence, if there are m obstacles, we will need $\frac{m}{k}$ such tours before the region is completely clean. On the other hand, Lemma 3 no longer holds (see Figure 15) since the boundary area can increase with time. However the boundary is always bounded above by the area. We now make the following conjecture :

Conjecture 1. Assume that k agents start at some boundary point of a non-simple connected region F_0 with s obstacles in it, and work according to the **CLEAN-WITH-OBSTACLES** algorithm, and denote by $t_{success}(k)$ the time needed for this group to clean F_0. Then it holds that :

$$\max\left\{2k, \left\lceil\frac{s_0}{k}\right\rceil, 2l(F_0)\right\} \leq t_{success}(k) \leq 8 \cdot s_0\left(\frac{W(F_0)}{k} + \left\lceil\frac{s}{k}\right\rceil\right) + 2k \quad (5)$$

where s_0,$l(F_0)$ and $W(F_0)$ denote the free area, length and width of F_0, respectively, and s is the number of obstacles in F_0.

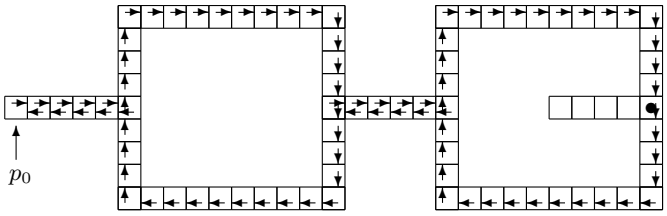


Fig. 13. An example of agents' odoring used in section 6 for cleaning regions with obstacles. The figure demonstrates the labels on the region's tiles after an agent reached the *useless* state, in which it did not clean any tile in its last tour. Note that any of the tiles labeled with one direction only, can be cleaned while still preserving the region's connectivity. Note that from those tiles, only tiles with a neighbor which is not labeled with any direction can not be cleaned, since it is indeed a critical point. Such a point is marked in the figure by a black circle.

Notice though, that the completion of the cleaning process is still guaranteed.

7 Experimental Results

A computer simulation, implementing the **CLEAN** protocol, was constructed. The algorithm was ran on several shapes of regions and for numbers of agents varying from 1 to 20. See Figure 14 - 15 for some examples of the evolution of the layout with time. The gray level of each pixel designates the index of the agent that actually cleaned this point. The right side of Figure 14 shows the same region and number of agents as in the left side of this Figure, but with randomly chosen initial locations at the corners. It can be seen that the dirty region is cleaned in a similar way. It should be said here that all the theory we developed in the previous sections (up to a small additive constant) applies to the case where the agents are initially located in randomly selected points on the boundary of F_0 (rather than starting from p_0). Figure 15 shows the evolution of the **CLEAN-WITH-OBSTACLES** algorithm for the same shapes with four additional obstacles in each, with 2 agents (left) and 10 agents (right).

Figure 16 depicts the timing results (as produced by computer simulations) describing the cleaning time as a function of the number of agents, compared to the theoretical bounds presented in previous sections (the cleaning time is normalized by the initial area of the dirty region). In Figure 17 we show the results for the same figures with additional obstacles, together with the conjectured theoretical bounds. All the figures include their appropriate statistical values.

Additional result are presented in Figure 18, comparing the various efficiency ratios obtained for initial dirty regions of various sizes. This is examined even further by Figure 19, which shows the behavior of $\frac{t_{success}(50)}{t_{success}(1)}$ namely, the ratio between the cleaning

time of 50 agents and the cleaning time required for a single agent. The change in this ratio can be seen, hinting the fact that as the initial region is larger, the addition of more agents can more efficiently decrease the cleaning time of the agents.

8 Discussion and Conclusion

Our cooperative **CLEAN**ing algorithm can be considered as a case of social behavior in the sense of [Sho1], where one induces multi-agent cooperation by forcing the agents to obey some simple "social" guidelines. This raises the question what happens if one agent malfunctions. We have shown that if less than k agents stop, the others will take over their responsibilities. But what if some agents start to cheat ? Such adversaries will have catastrophic consequences, since a crazy agent may clean a critical point and disconnect the dirty region.

Another question of interest is the resolution of collisions between agents. In the **CLEAN** protocol we resolve such a problem by giving each agent a priority measure depending on his previous location. However, it is an interesting open question whether a coin flipping might be better here.

The cleaning problem discussed is related to the geometric problem of pocket-machining, see [Hel1] for details. An interesting problem of cleaning and maintenance of a system of pipes by an autonomous agent is discussed in [Neu1]. The importance of cleaning hazardous waste by agents is described in [Hed1].

Our approach is that cleaning is always done at the boundary. It is possible that a better efficiency will be achieved using other approaches:

1. Given that several neighbors are dirty, visit the non-critical ones first (even if not on the boundary). This approach is quite efficient for one agent, but can be a mess for several agents.
2. Once entering a large "room" (that is - upon passing from a critical area to a non-critical one) - designate the entrance by a special type of token, so that other agents will enter only in case there is no other work to do. This approach guarantees that the agents will be distributed between the large rooms of the F-configuration. This is attractive if the region has such rooms of quite similar areas.
3. Quite a different idea is to divide the work into two phases - the first one of "distribution" - the agents locate themselves uniformly around the area. Then, in the second phase, each agent cleans around his "center". If the distribution is appropriate, there will be minimum of interactions between agents in the second phase.

We use the dirt on the floor as a means of inter-agent communication, but other ways for communication between agents have been suggested. One is to use heat trails for this end, as was reported in [Rus1]. In [Ste1], self-organization is achieved among a group of lunar robots that have to explore an unknown region, and bring special rock-samples back to the mother-spaceship, by programming each robot to drop a crumb at each point he visits and walk around at random with a bias toward the negative gradient of crumb concentration.

Another question of interest is how to guarantee covering, at least with high probability, without using any external signs, using only the inter-agent collisions as an indicator for a good direction to proceed.

It is of interest to notice here that an off-line version of the problem, that is : finding the shortest path that visits all grid points in F, where F is completely known in advance, is *NP-hard* even for a single agent. It is a corollary of the fact that Hamilton path in a non-simple grid-graph is *NP-complete* [Ita1].

In summary, we would like to cite a statement made by a scientist after watching an ant making his laborious way across a wind-and-wave-molded beach[Sim1]:

> An ant, viewed as a behaving system, is quite simple. The apparent complexity of its behavior over time is largely a reflection of the environment in which it finds itself.

Such a point of view, as well as the results of our simulations and analysis, make us believe that even simple, ant-like creatures, yield very interesting, adaptive and quite efficient goal oriented behavior.

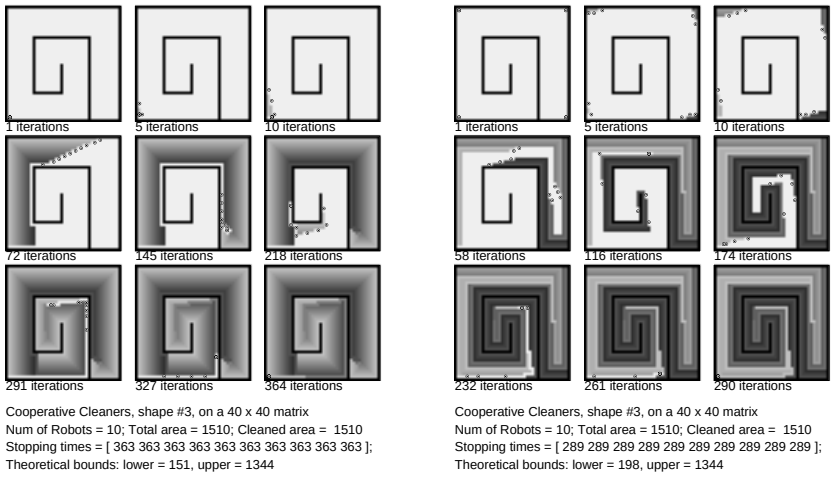


Fig. 14. Cooperative cleaners: maze, 10 agents (left), and 10 agents with random initial locations on the boundary (right).

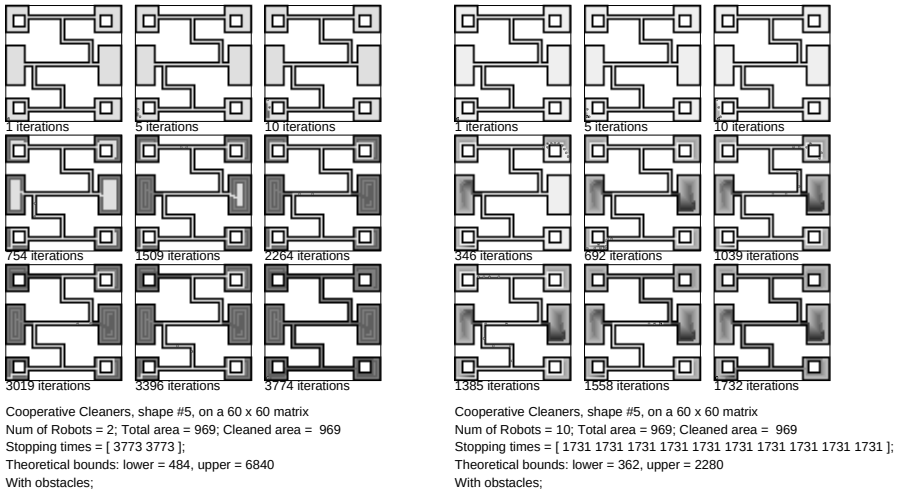
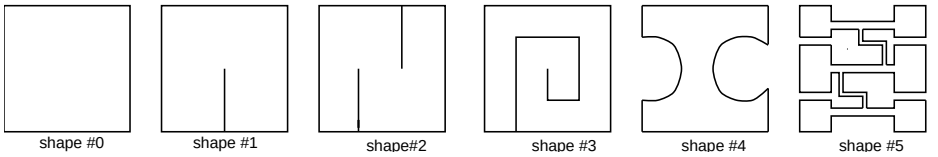
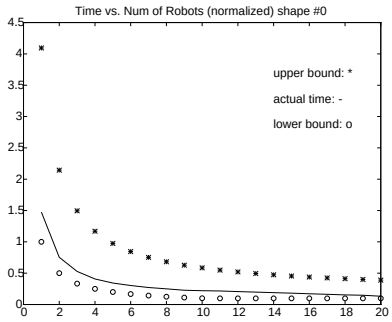
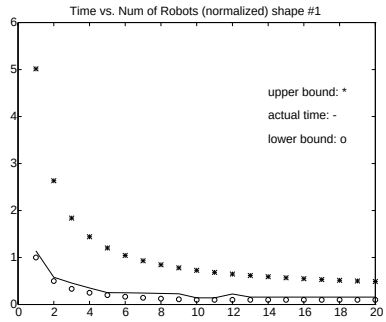
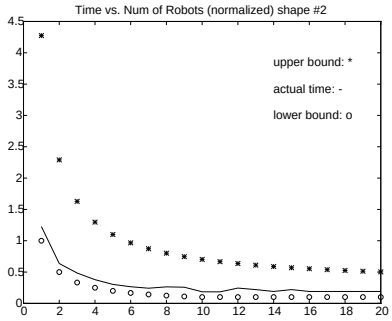


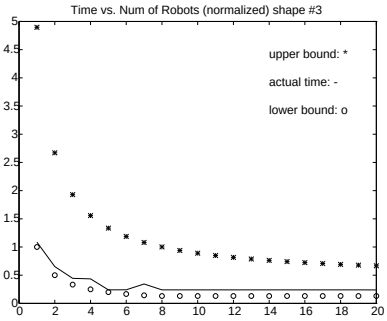
Fig. 15. Cooperative cleaners: 2 agents, 6 rooms with obstacles (left), 10 agents, 6 rooms with obstacles (right).

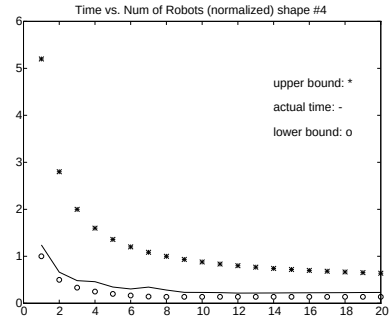












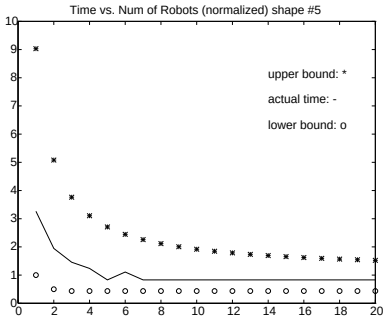
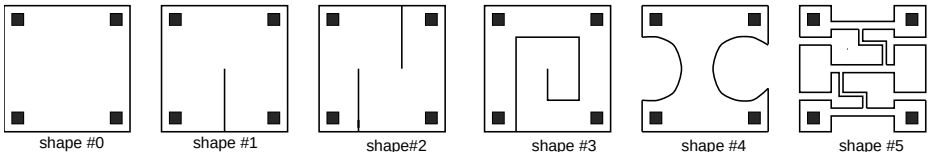
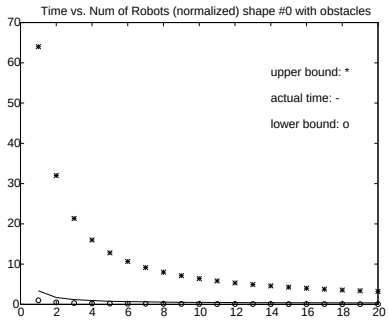
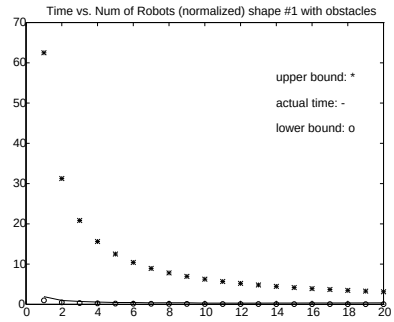
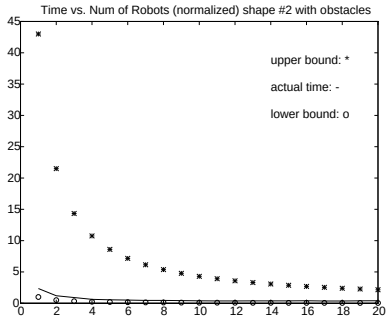


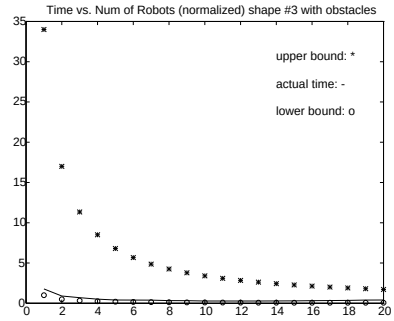
Fig. 16. Various shapes tested in **CLEAN** simulations and their normalized cleaning time $t(k) = \frac{t_{success}(k)}{s_0}$ as a function of the number of agents, k, for each region, together with the lower and upper bounds according to Theorem 2. For a confidence level of 95%, based on at least 10 simulation instances per each dirty region, the appropriate confidence interval is less than $\pm 6\%$.

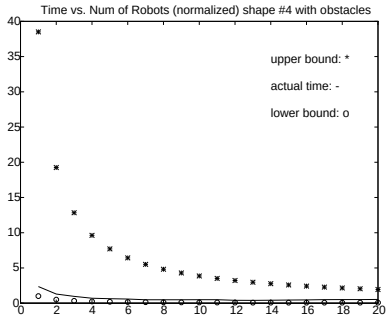












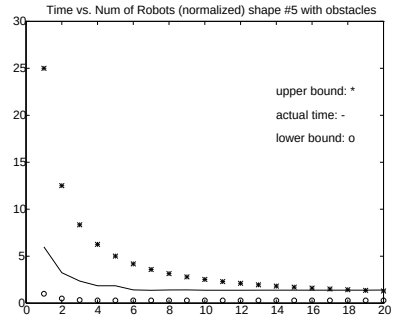


Fig. 17. Various shapes tested in **CLEAN-WITH-OBSTACLES** simulations and their normalized cleaning time $t(k) = \frac{t_{success}(k)}{s_0}$ as a function of the number of agents, k, for each region, together with the lower and upper bounds according to Theorem 2. For a confidence level of 95%, based on at least 10 simulation instances per each dirty region, the appropriate confidence interval is less than $\pm 10\%$.

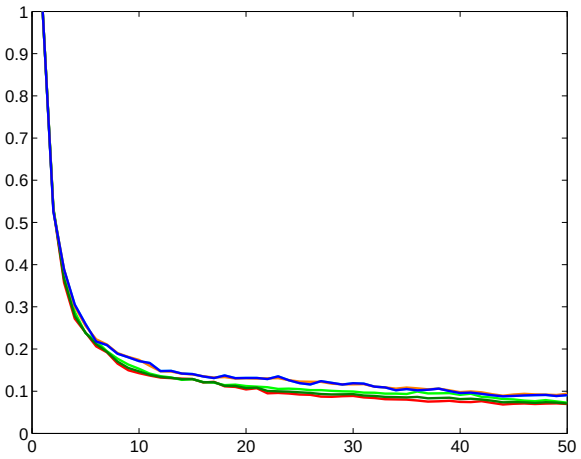


Fig. 18. Cooperative cleaners: from 1 to 50 agents, dirty regions of sizes 600 to 1500 tiles, 10 simulations per region's size, cleaning time normalized by the cleaning time of a single agent. Note that as the initial dirty area is larger, the ratio between $t_{success}(k)$ and $t_{success}(1)$ increases. In fact, comparing the time ratios of regions of sizes 600 and 1500 tiles, the difference reaches 30%. For a confidence level of 95% the confidence interval is less than $\pm 15\%$.

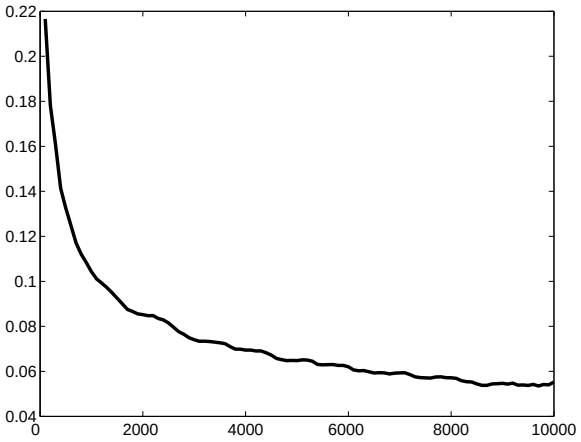


Fig. 19. Cooperative cleaners: graph demonstrates the behavior of $\frac{t_{success}(50)}{t_{success}(1)}$ for dirty regions of sizes 100 to 10000 tiles. 10 simulations were done per region's size. Note how as the initial dirty area gets larger $\frac{t_{success}(50)}{t_{success}(1)}$ decreases. The minimal value possible is of course $\frac{1}{50} = 0.02$. For a confidence level of 90% the confidence interval is less than $\pm 10\%$.

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