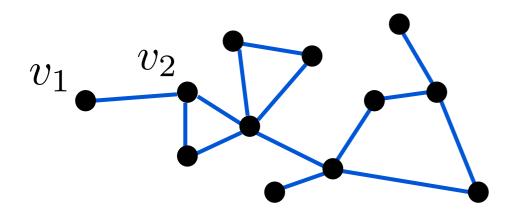
Applied Machine Learning

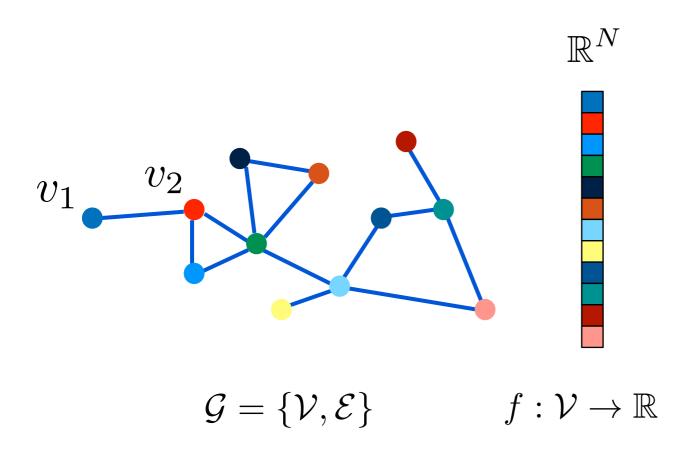
Graph machine learning - Part II

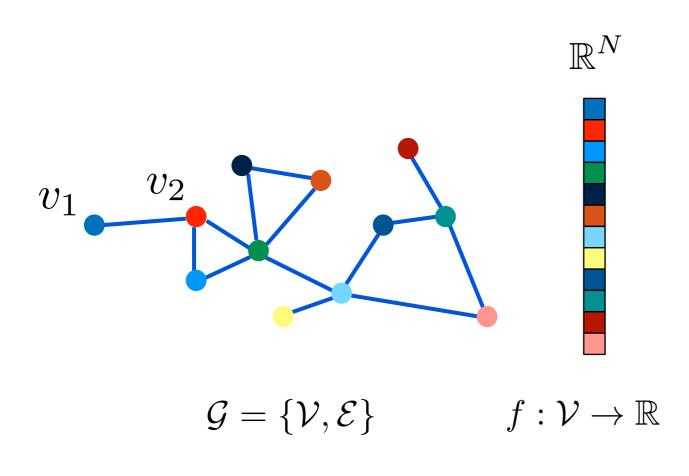
Xiaowen Dong
Department of Engineering Science





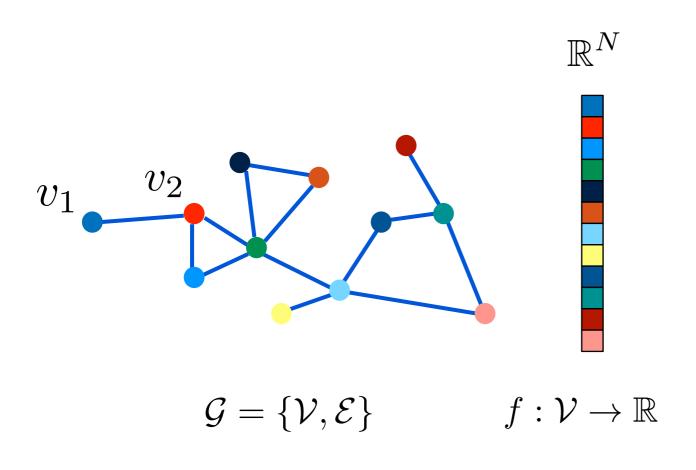
$$\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$$





- key concepts
 - smoothness

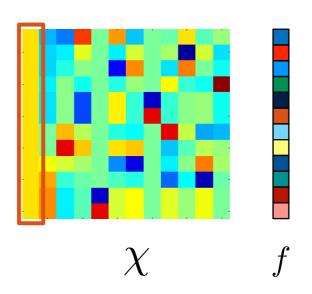
$$f^{T}Lf = \frac{1}{2} \sum_{i,j=1}^{N} W_{ij} (f(i) - f(j))^{2}$$

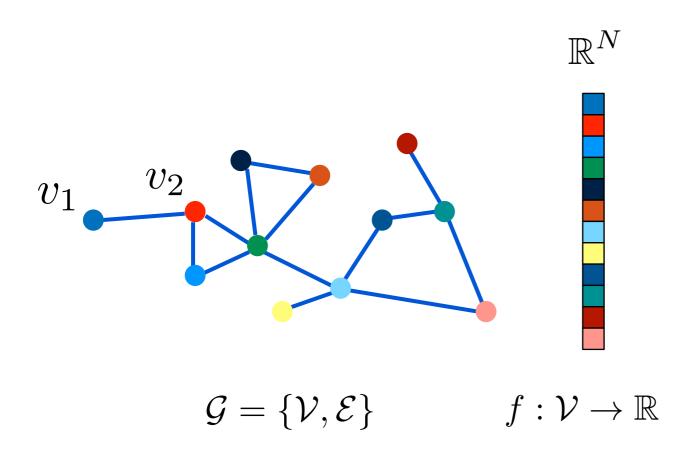


- key concepts
 - smoothness

$$f^{T}Lf = \frac{1}{2} \sum_{i,j=1}^{N} W_{ij} (f(i) - f(j))^{2}$$

- Fourier-like analysis

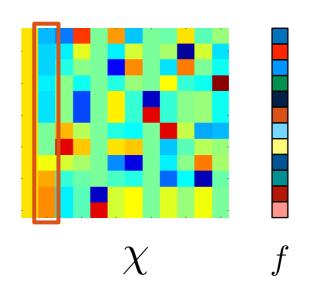


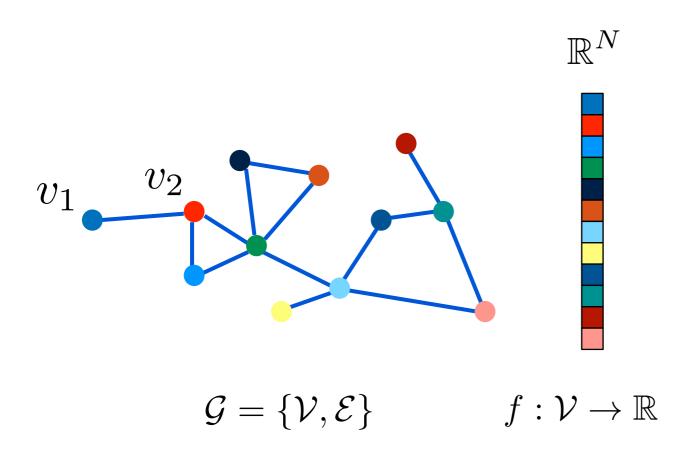


- key concepts
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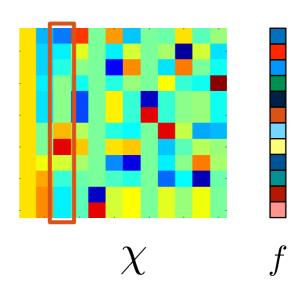




- key concepts
 - smoothness

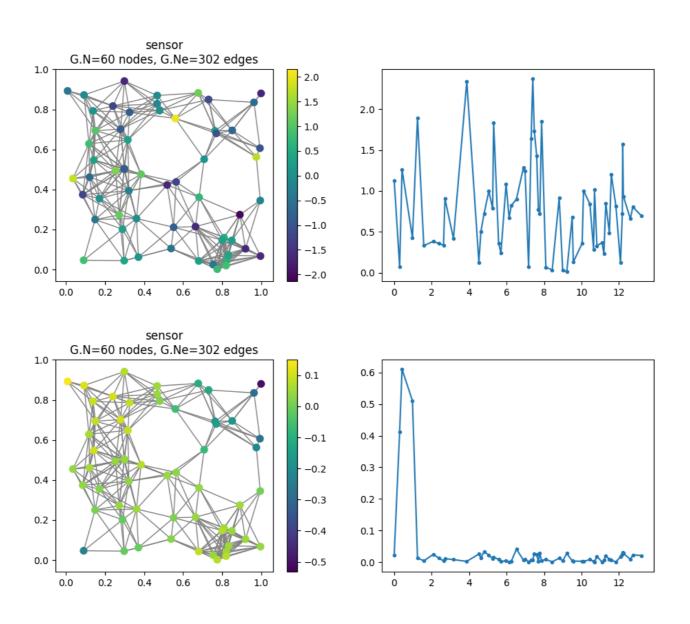
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- Fourier-like analysis

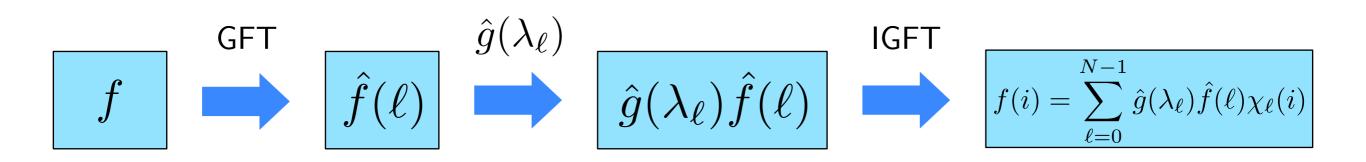


Graph Fourier transform

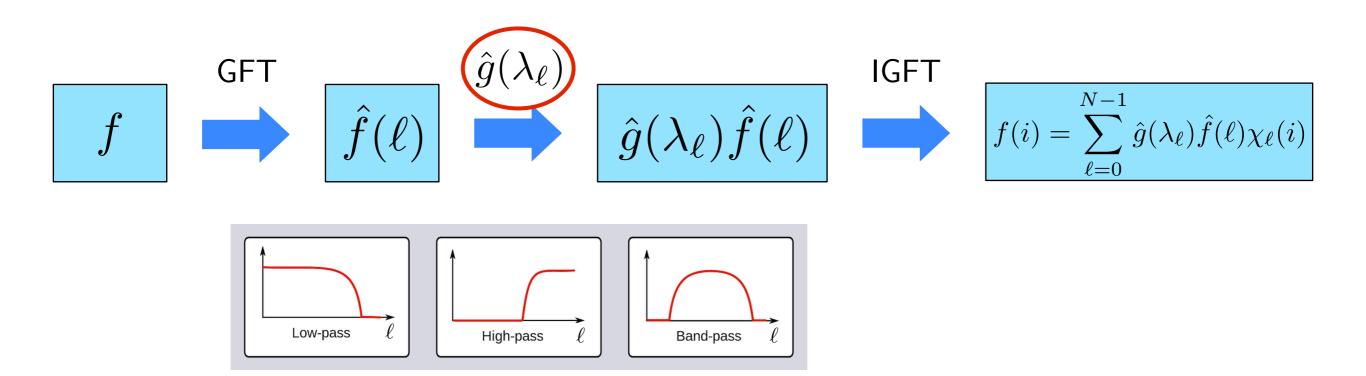
$$\hat{f}(\ell) = \langle \chi_{\ell}, f \rangle : egin{bmatrix} | \chi_0 & \cdots & \chi_{N-1} | f \\ | \chi_0 & \cdots & \chi_{N-1} | f \\ | \chi_0 & \cdots & \chi_{N-1} | f \\ | \chi_0 & \cdots & \chi_{N-1} | f \\ | \chi_0 & \cdots & \chi_{N-1} | f \\ | \chi_0 & \cdots & \chi_{N-1} | \chi_{N-1} | f \\ | \chi_0 & \cdots & \chi_{N-1} | \chi_{N-1$$



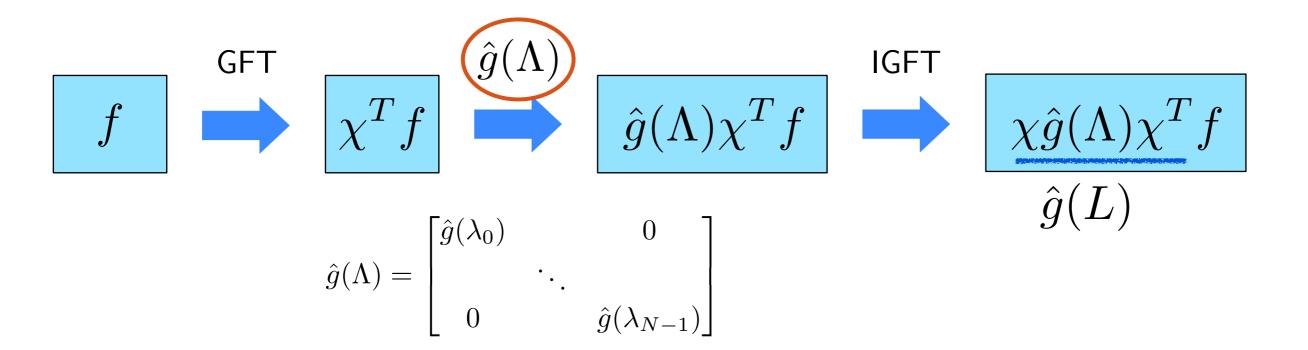
Graph spectral filtering

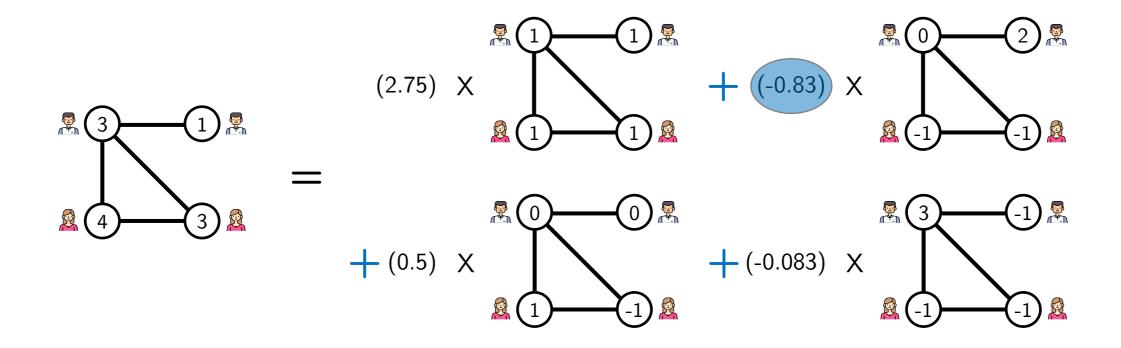


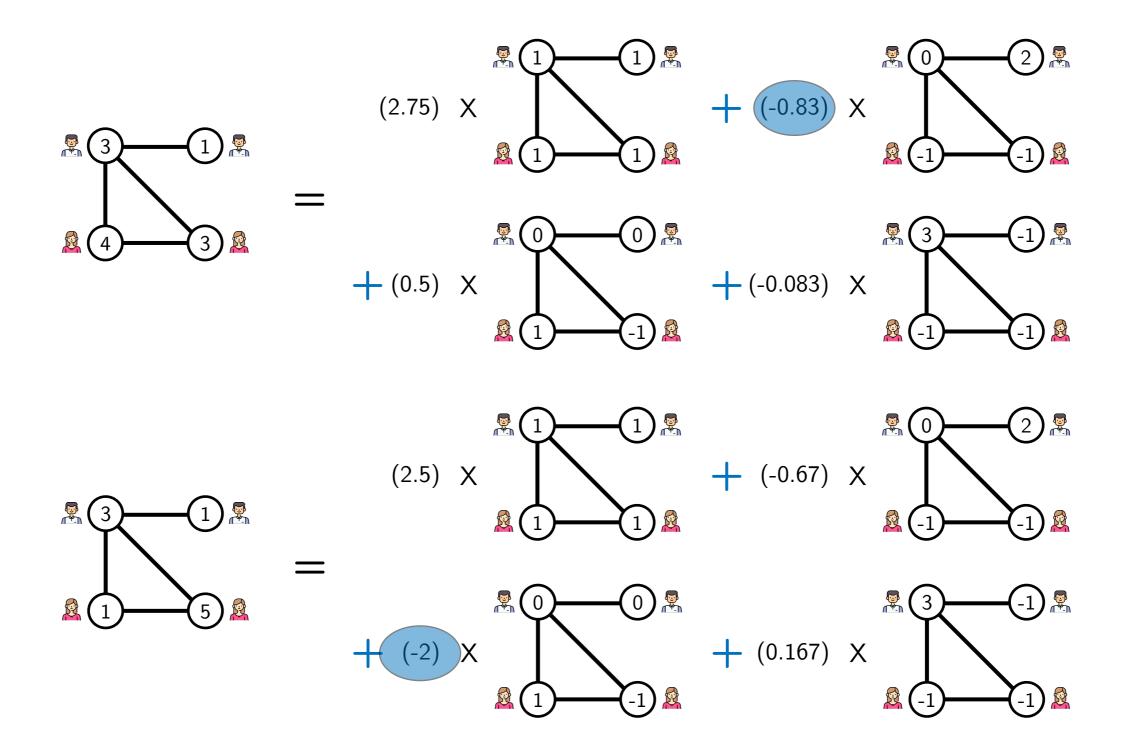
Graph spectral filtering

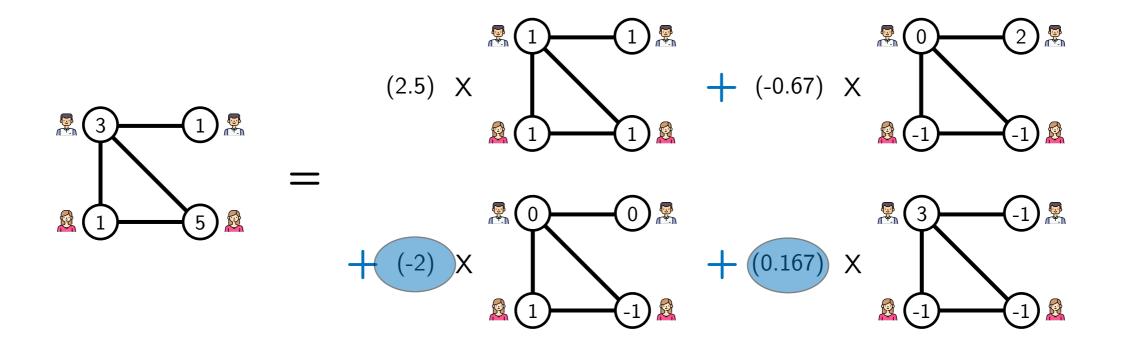


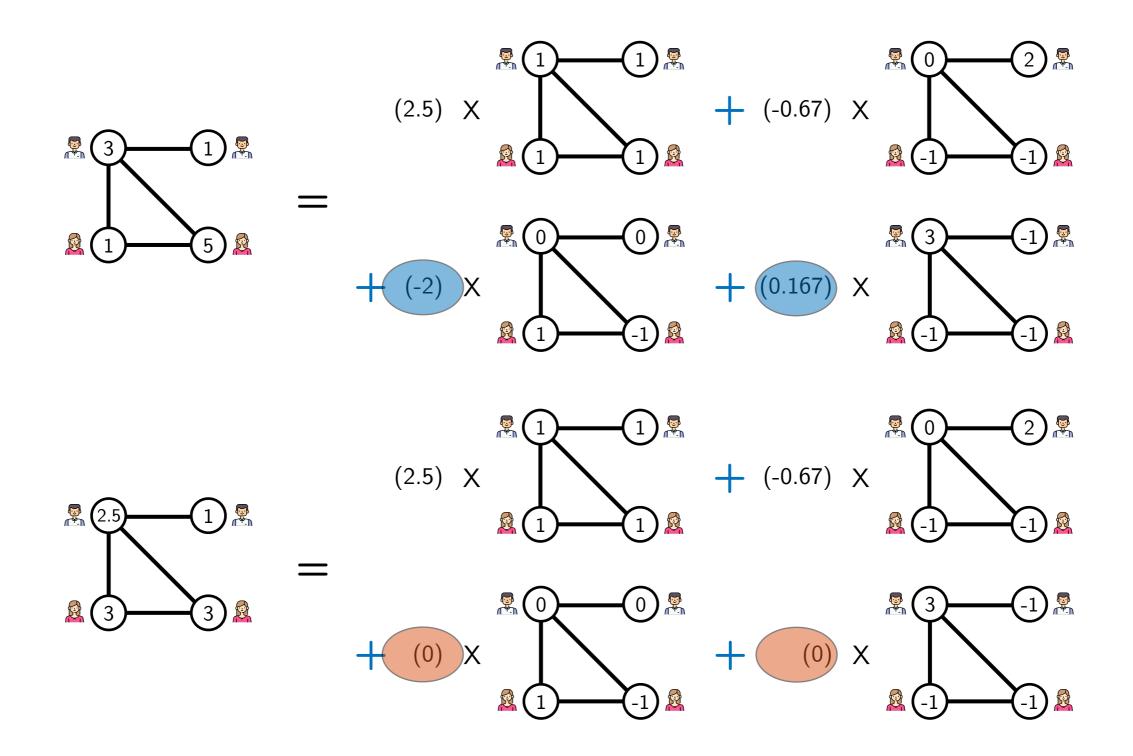
Graph spectral filtering





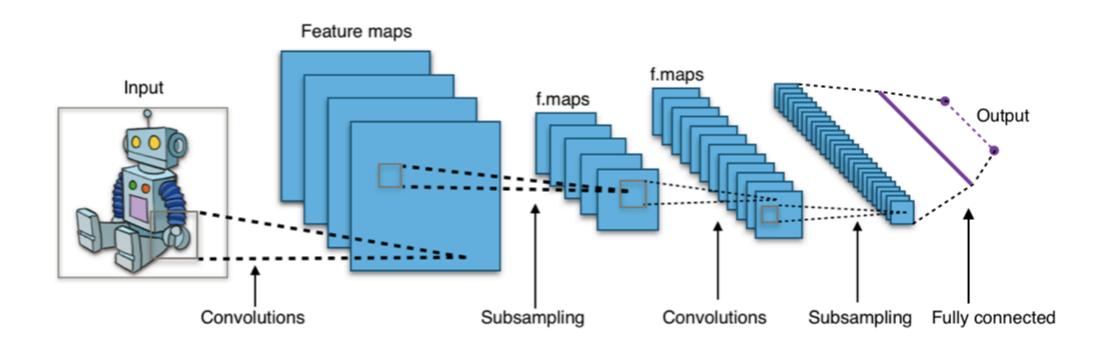






Graph Neural Networks

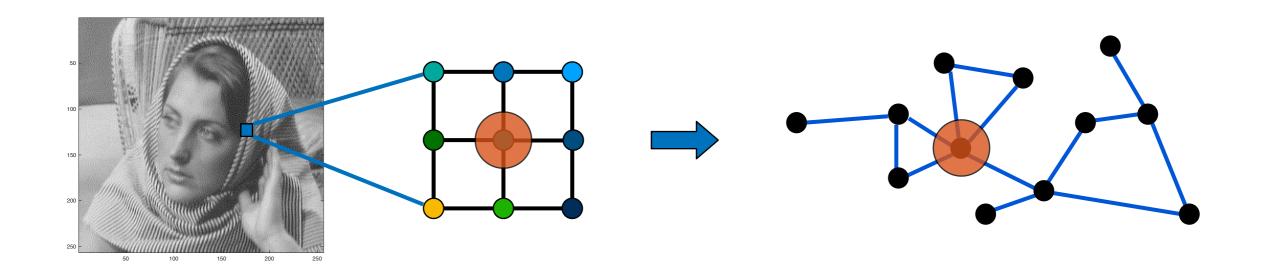
CNNs exploit structure within data



checklist

- convolution: translation equivariance
- localisation: compact filters (independent of sample dimension)
- multi-scale: compositionality
- **efficiency:** $\mathcal{O}(N)$ computational complexity

CNNs on graphs?



checklist

- convolution: how to do it on graphs?
- localisation: what's the notion of locality?
- multi-scale: how to down-sample on graphs?
- **efficiency:** how to keep the computational complexity low?

classical convolution

time domain

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau$$

30	3	2_2	1	0
02	0_2	1_0	3	1
30	1,	2	2	3
2	0	0	2	2
2	0	0	0	1

12.0	12.0	17.0
10.0	17.0	19.0
9.0	6.0	14.0

classical convolution

time domain

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau$$

30	3	2°_2	1	0
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classical convolution

time domain

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau$$



frequency domain

$$\widehat{(f * g)}(\omega) = \hat{f}(\omega) \cdot \hat{g}(\omega)$$

30	3	2_{2}	1	0
0_2	0_2	1_{0}	3	1
30	1,	22	2	3
2	0	0	2	2
2	0	0	0	1

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classical convolution

convolution on graphs

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$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau$$



frequency domain

$$\widehat{(f * g)}(\omega) = \hat{f}(\omega) \cdot \hat{g}(\omega)$$

graph spectral domain

$$\widehat{(f*g)}(\lambda) = ((\chi^T f) \circ \hat{g})(\lambda)$$

classical convolution

time domain

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau$$



frequency domain

$$\widehat{(f * g)}(\omega) = \widehat{f}(\omega) \cdot \widehat{g}(\omega)$$

convolution on graphs

spatial (node) domain

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



graph spectral domain

$$\widehat{(f * g)}(\lambda) = ((\chi^T f) \circ \hat{g})(\lambda)$$

classical convolution

time domain

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frequency domain

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convolution on graphs

spatial (node) domain

$$f*g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$
 convolution = filtering



graph spectral domain

$$\widehat{(f * g)}(\lambda) = ((\chi^T f) \circ \hat{g})(\lambda)$$

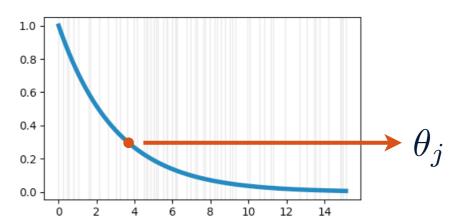
A non-parametric filter

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



learning a non-parametric filter:

$$\hat{g}_{\theta}(\Lambda) = \operatorname{diag}(\theta), \ \theta \in \mathbb{R}^{N}$$



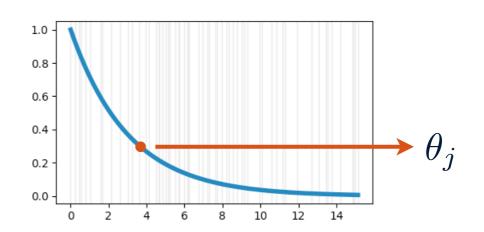
A non-parametric filter

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



learning a non-parametric filter:

$$\hat{g}_{\theta}(\Lambda) = \operatorname{diag}(\theta), \ \theta \in \mathbb{R}^{N}$$



- convolution expressed in the graph spectral domain
- no localisation in the spatial (node) domain
- computationally expensive

A parametric filter

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



parametric filter as polynomial of Laplacian

$$\hat{g}_{\theta}(\lambda) = \sum_{j=0}^{K} \theta_{j} \lambda^{j}, \ \theta \in \mathbb{R}^{K+1} \qquad \qquad \hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_{j} L^{j}$$



$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_{j} L^{j}$$

A parametric filter

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parametric filter as polynomial of Laplacian

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$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_{j} L^{j}$$

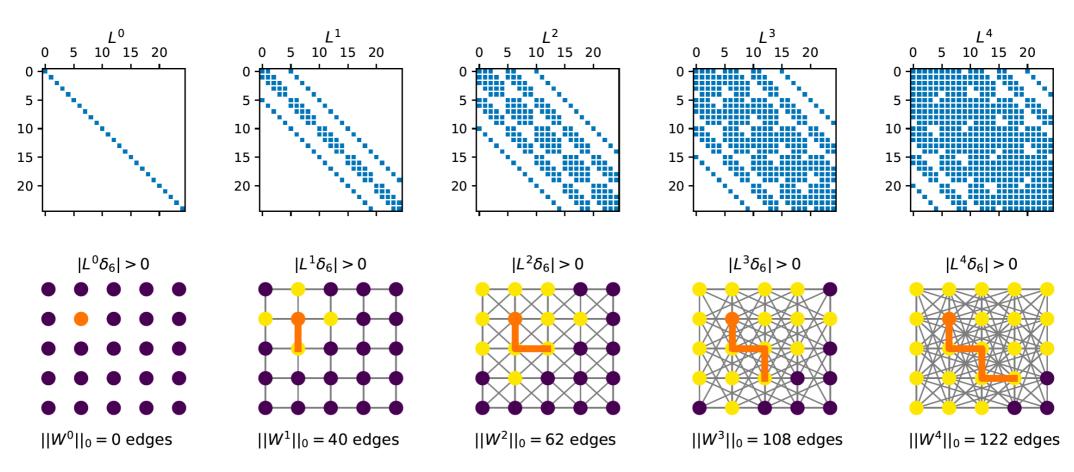


$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_{j} L^{j}$$

what do powers of graph Laplacian capture?

Powers of graph Laplacian

L^k defines the k-neighborhood



Localization: $d_{\mathcal{G}}(v_i, v_i) > K$ implies $(L^K)_{ij} = 0$

(slide by Michaël Deferrard)

A parametric filter

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



parametric filter as polynomial of Laplacian

$$\hat{g}_{\theta}(\lambda) = \sum_{j=0}^{K} \theta_{j} \lambda^{j}, \ \theta \in \mathbb{R}^{K+1} \qquad \qquad \hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_{j} L^{j}$$



$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_{j} L^{j}$$

A parametric filter

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$

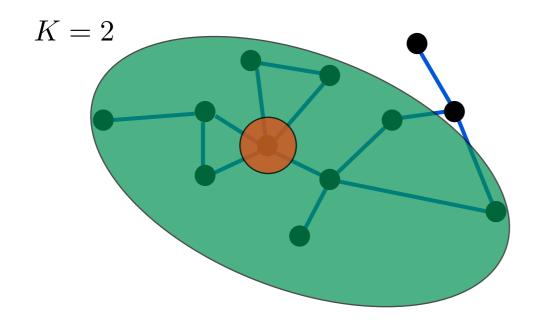


parametric filter as polynomial of Laplacian

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$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_j L^j$$



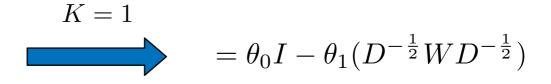
- localisation within K-hop neighbourhood
- efficient computation via recursive multiplication with L

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$

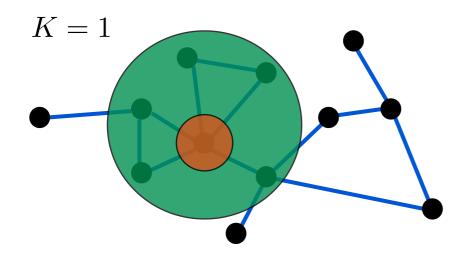


simplified parametric filter

$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_j L^j$$



(localisation within 1-hop neighbourhood)

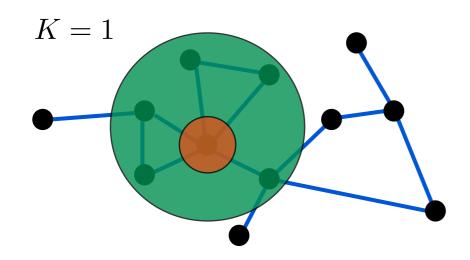


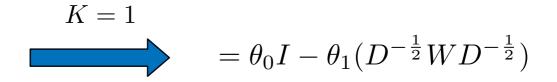
$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



simplified parametric filter

$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_j L^j$$





(localisation within 1-hop neighbourhood)

$$\alpha = \theta_0 = -\theta_1$$

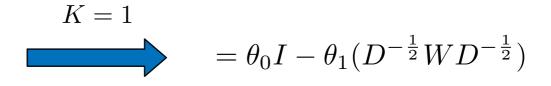
$$= \alpha (I + D^{-\frac{1}{2}} W D^{-\frac{1}{2}})$$

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$

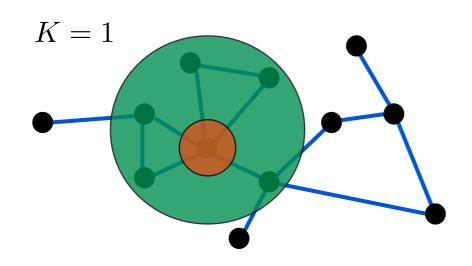


simplified parametric filter

$$\hat{g}_{\theta}(L) = \sum_{j=0}^{K} \theta_j L^j$$



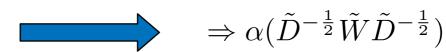
(localisation within 1-hop neighbourhood)



$$\alpha = \theta_0 = -\theta_1$$

$$= \alpha (I + D^{-\frac{1}{2}} W D^{-\frac{1}{2}})$$

renormalisation



$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



simplified parametric filter

$$\hat{g}_{\alpha}(L) = \alpha(I + D^{-\frac{1}{2}}WD^{-\frac{1}{2}})$$

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$

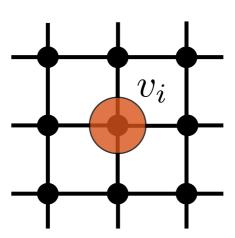


simplified parametric filter

$$\hat{g}_{\alpha}(L) = \alpha(I + D^{-\frac{1}{2}}WD^{-\frac{1}{2}})$$



$$y_i = \alpha f_i + \alpha \frac{1}{\sqrt{d_i}} \sum_{j:(i,j)\in\mathcal{E}} w_{ij} \frac{1}{\sqrt{d_j}} f_j$$



A simplified parametric filter

$$f * g = \chi \hat{g}(\Lambda) \chi^T f = \hat{g}(L) f$$



simplified parametric filter

$$\hat{g}_{\alpha}(L) = \alpha(I + D^{-\frac{1}{2}}WD^{-\frac{1}{2}})$$

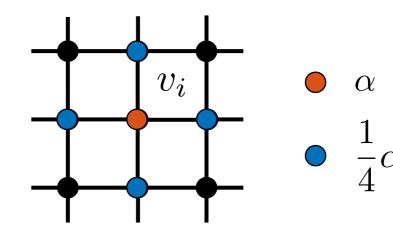


$$y_i = \alpha f_i + \alpha \frac{1}{\sqrt{d_i}} \sum_{j:(i,j)\in\mathcal{E}} w_{ij} \frac{1}{\sqrt{d_j}} f_j$$



unitary edge weights

$$y_i = \alpha f_i + \frac{1}{4} \alpha \sum_{j:(i,j)\in\mathcal{E}} f_j$$



A simplified parametric filter

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simplified parametric filter

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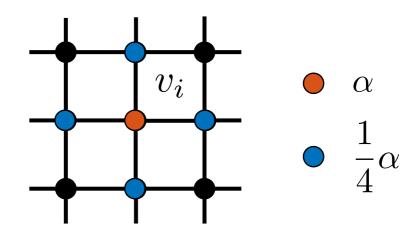


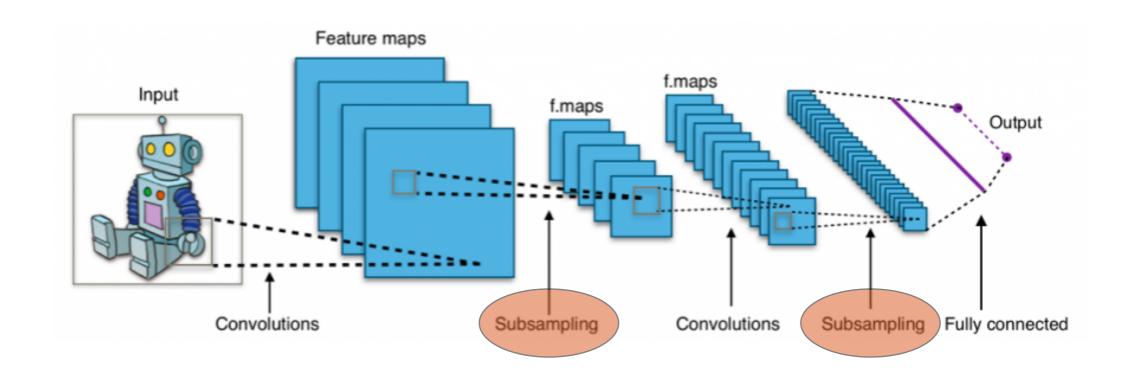
unitary edge weights

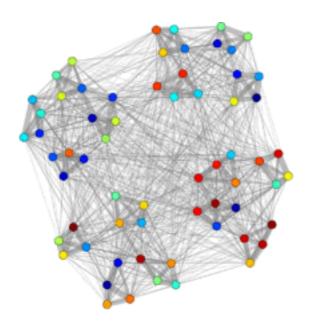
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30	3,	22	1	0
0_2	0_2	1_{0}	3	1
30	1,	2	2	3
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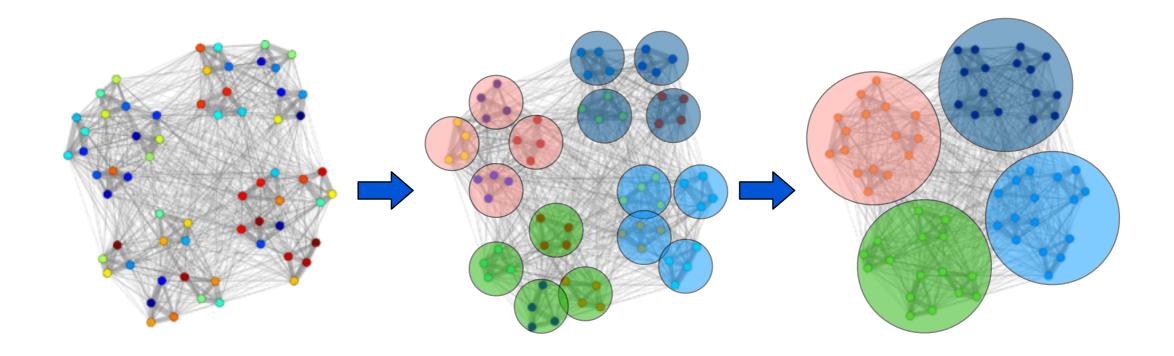
12.0	12.0	17.0
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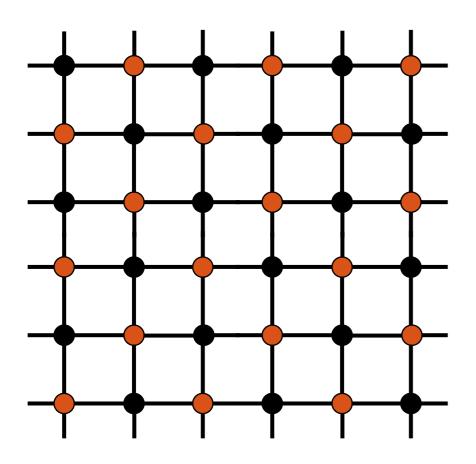




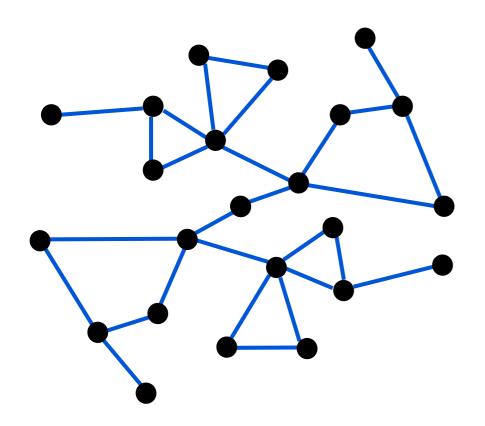
• pooling = downsampling on graphs, but how?



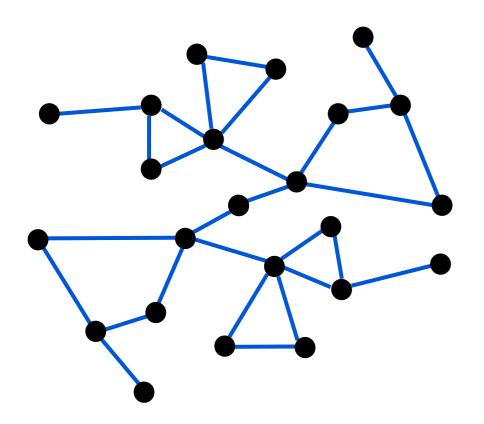
- pooling = downsampling on graphs, but how?
- natural idea: graph coarsening



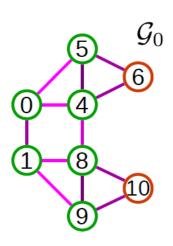
coarsening is straightforward on regular grids



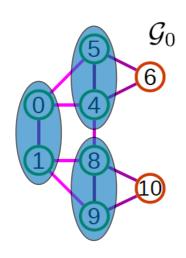
- coarsening is straightforward on regular grids
- not so much on irregular graphs



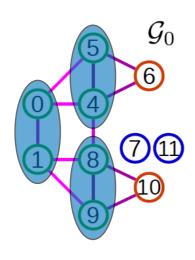
- coarsening is straightforward on regular grids
- not so much on irregular graphs
- can be achieved via node clustering
 - multi-level partitioning
 - roughly fixed downsampling factor (e.g., 2)
 - need for efficiency



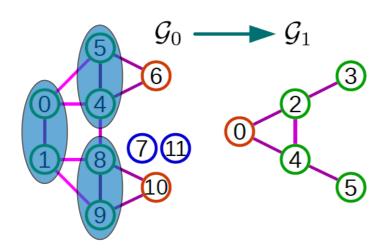
- pooling based on Graclus algorithm (Dhillon et al. 2007)
 - local greedy way of merging vertices: maximising $w_{ij}(1/d_i+1/d_j)$
 - adding artificial vertices to ensure two children for each node



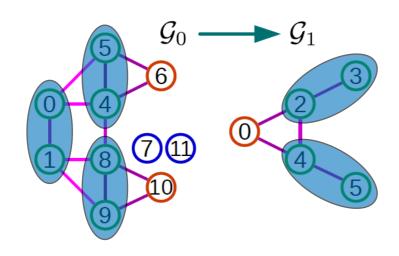
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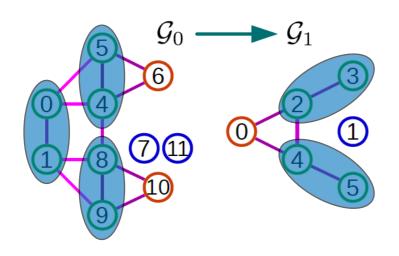
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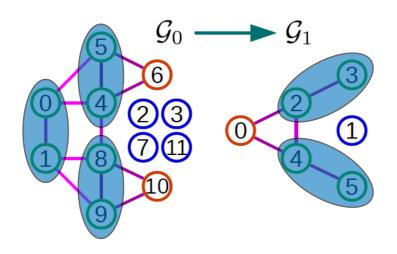
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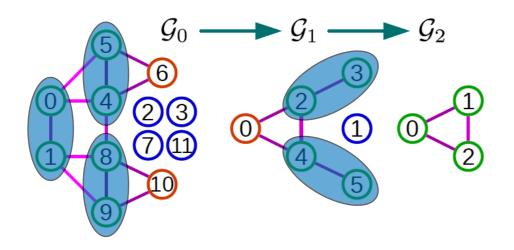
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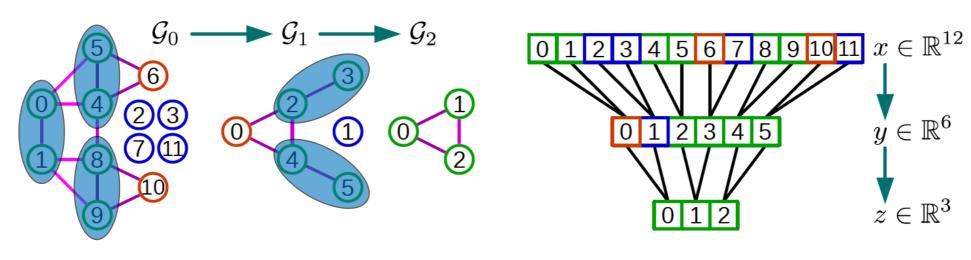
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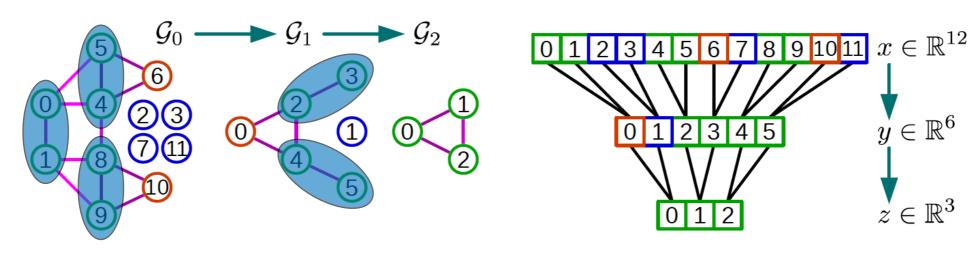


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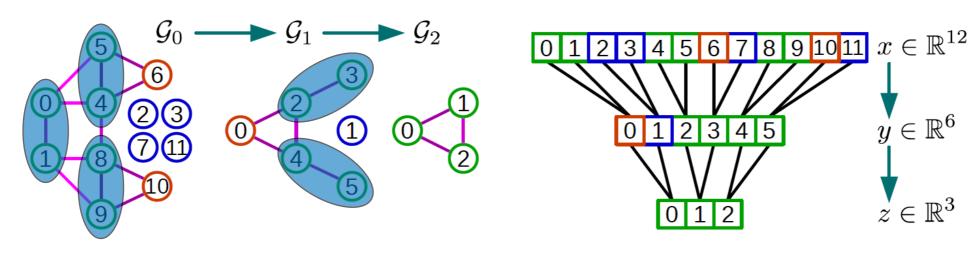
Defferrard et al. 2016

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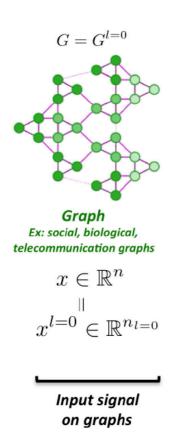
Defferrard et al. 2016

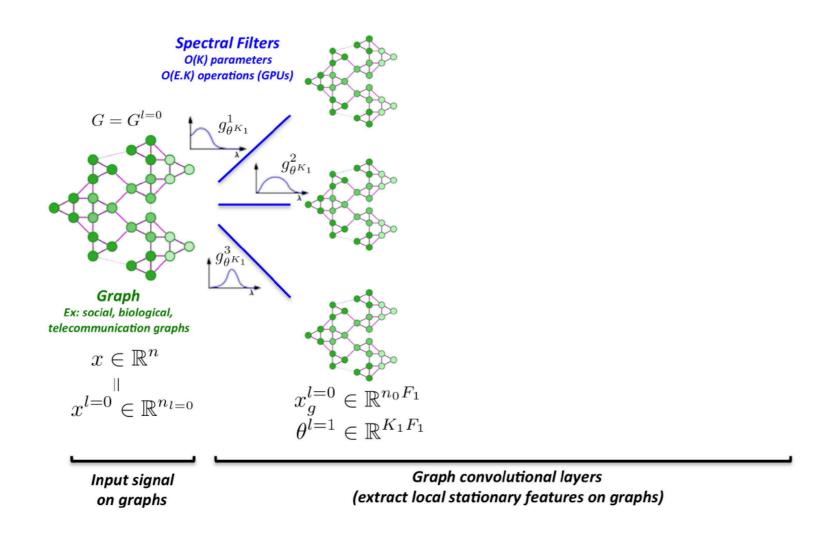
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 - 1D grid pooling: $[\max(0,1)\max(4,5,6)\max(8,9,10)]$

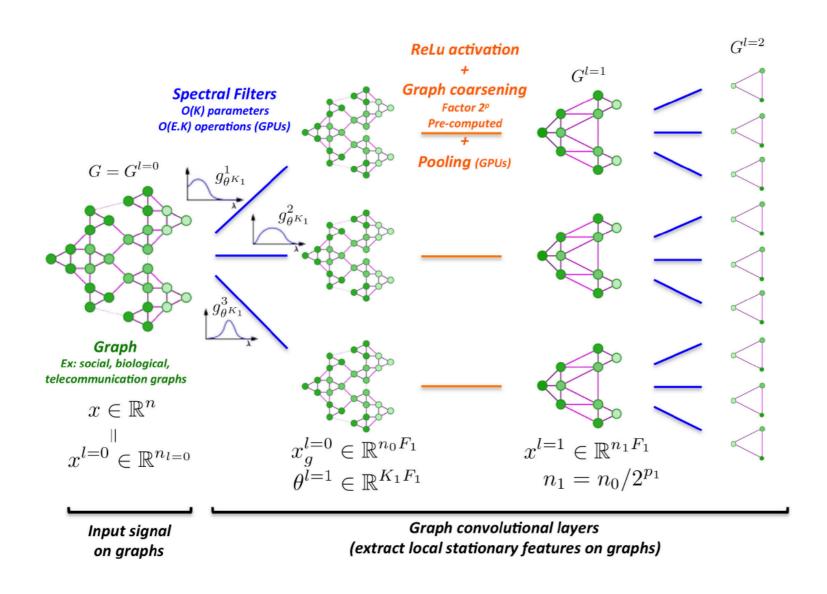


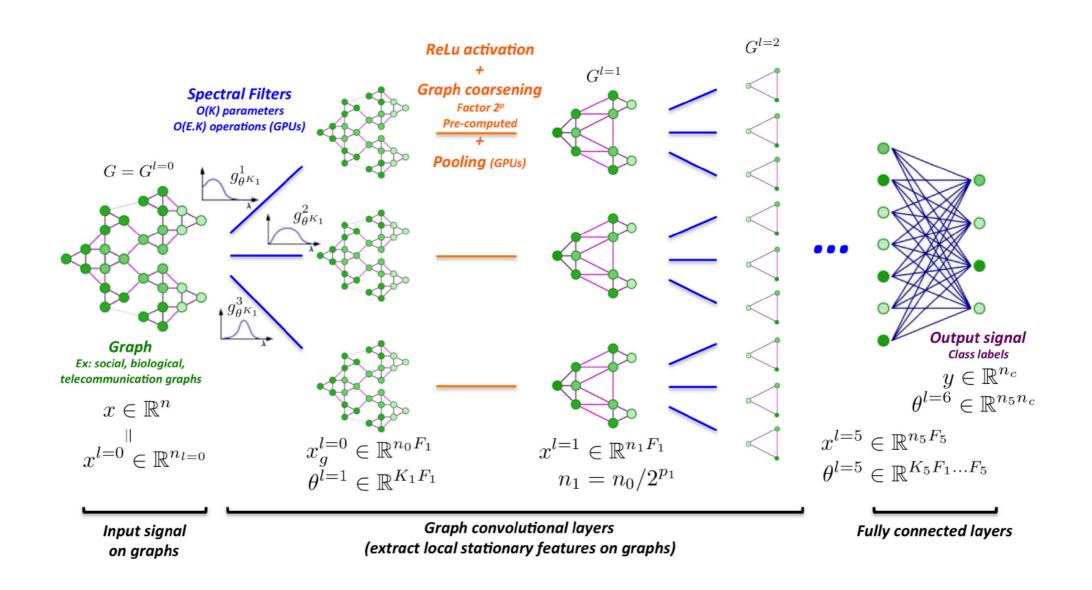
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 - local greedy way of merging vertices: maximising $w_{ij}(1/d_i+1/d_j)$
 - adding artificial vertices to ensure two children for each node
 - 1D grid pooling: [max(0,1) max(4,5,6) max(8,9,10)]
 - only based on graph (and no signal) information

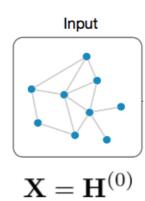




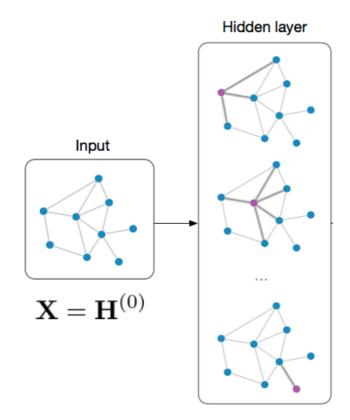




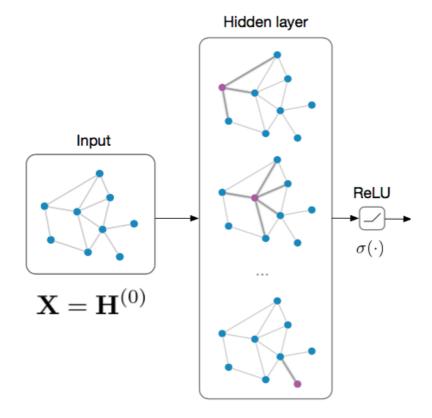
$$\hat{g}_{\theta^{(k+1)}}(L)\Big(\mathrm{ReLU}(\hat{g}_{\theta^{(k)}}(L)f)\Big)$$



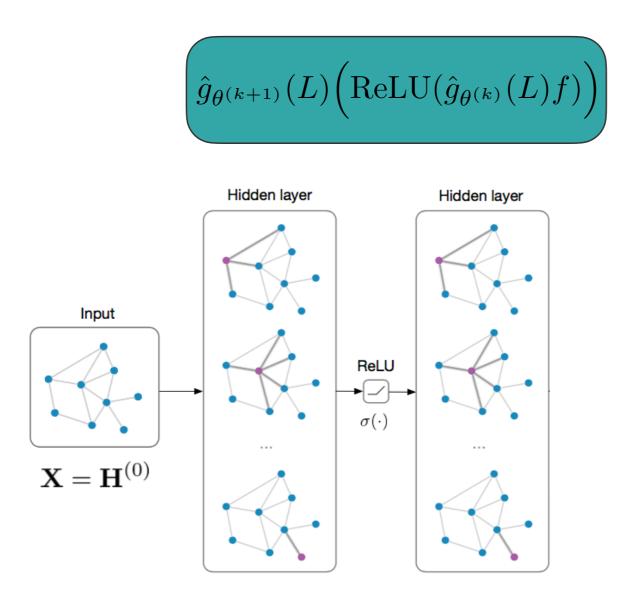
$$\hat{g}_{\theta^{(k+1)}}(L)\Big(\mathrm{ReLU}(\hat{g}_{\theta^{(k)}}(L)f)\Big)$$

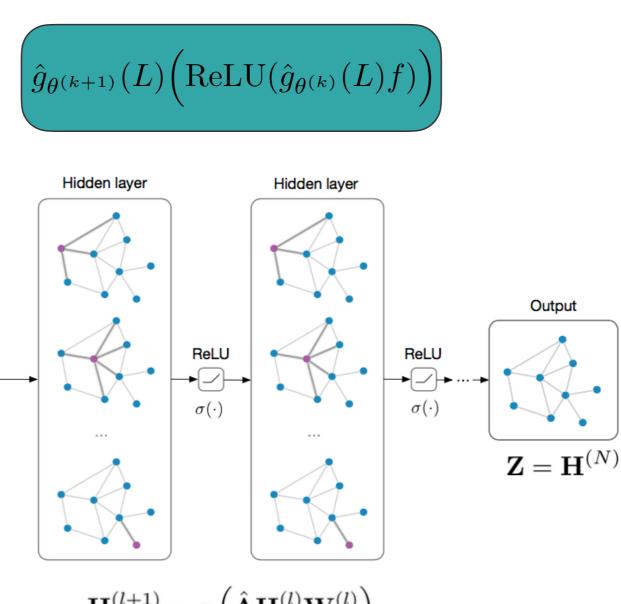






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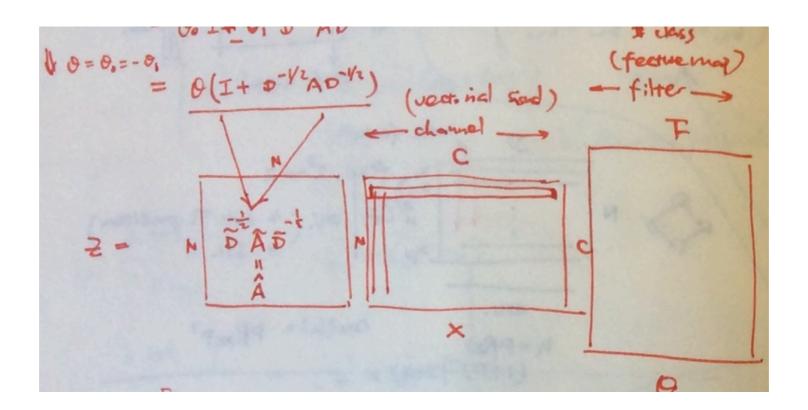
$$\mathbf{H}^{(l+1)} = \sigma \left(\hat{\mathbf{A}} \mathbf{H}^{(l)} \mathbf{W}^{(l)} \right)$$

21/30

Input

 $\mathbf{X} = \mathbf{H}^{(0)}$

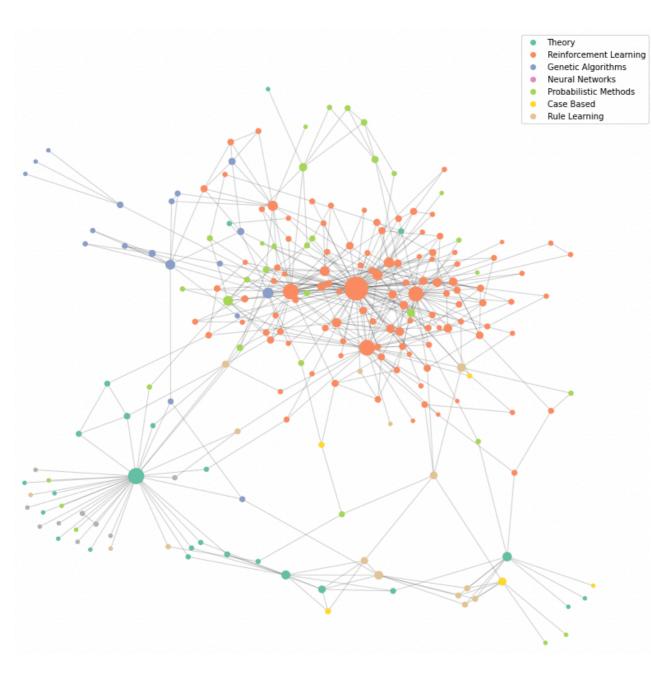
$$\left(\hat{g}_{ heta^{(k+1)}}(L)\Big(\mathrm{ReLU}(\hat{g}_{ heta^{(k)}}(L)f)\Big)\right)$$



$$\mathbf{H}^{(l+1)} = \sigma \left(\hat{\mathbf{A}} \mathbf{H}^{(l)} \mathbf{W}^{(l)} \right)$$

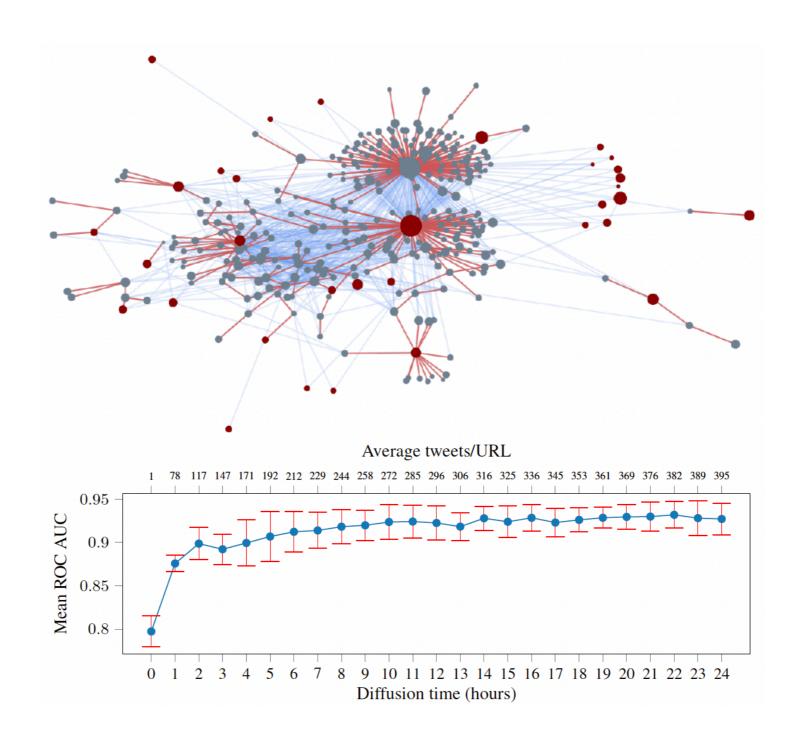
21/30

Application I: Document classification

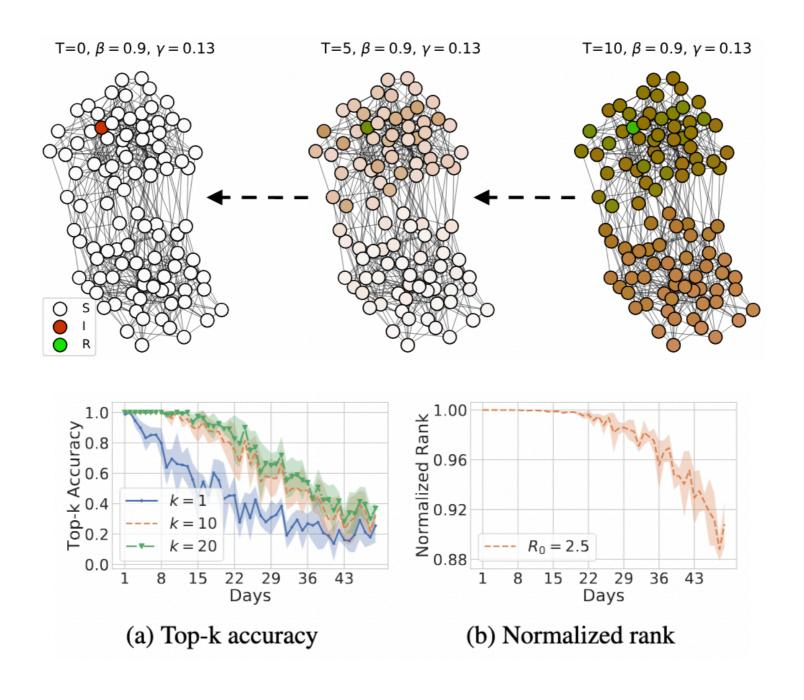


Method	Citeseer	Cora	Pubmed	NELL
ManiReg [3]	60.1	59.5	70.7	21.8
SemiEmb [28]	59.6	59.0	71.1	26.7
LP [32]	45.3	68.0	63.0	26.5
DeepWalk [22]	43.2	67.2	65.3	58.1
ICA [18]	69.1	75.1	73.9	23.1
Planetoid* [29]	64.7 (26s)	75.7 (13s)	77.2 (25s)	61.9 (185s)
GCN (this paper)	70.3 (7s)	81.5 (4s)	79.0 (38s)	66.0 (48s)
GCN (rand, splits)	67.9 ± 0.5	80.1 ± 0.5	78.9 ± 0.7	58.4 ± 1.7

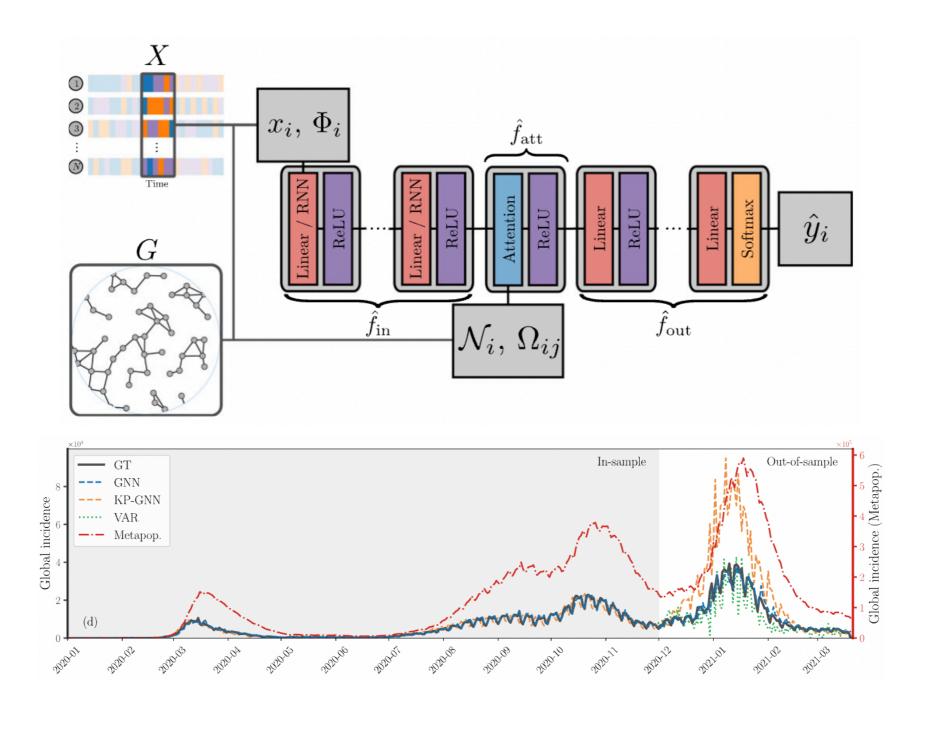
Application II: Fake news detection



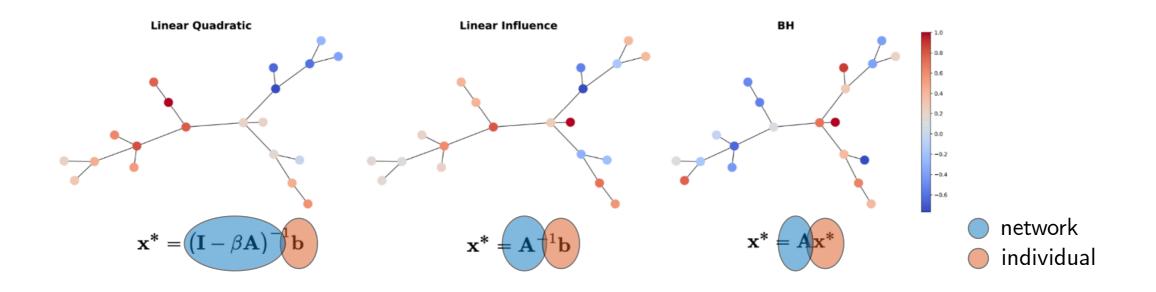
Application III: Finding patient zero



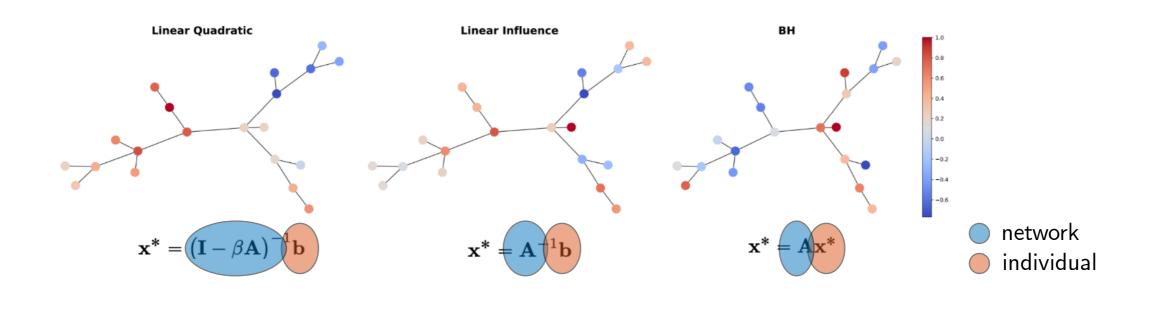
Application IV: Learning contagion dynamics

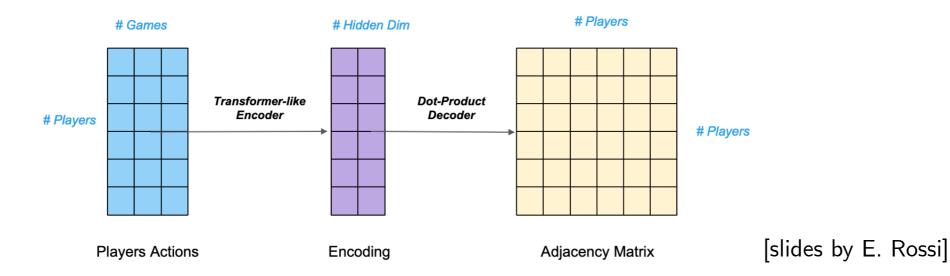


Application V: Learning social interactions

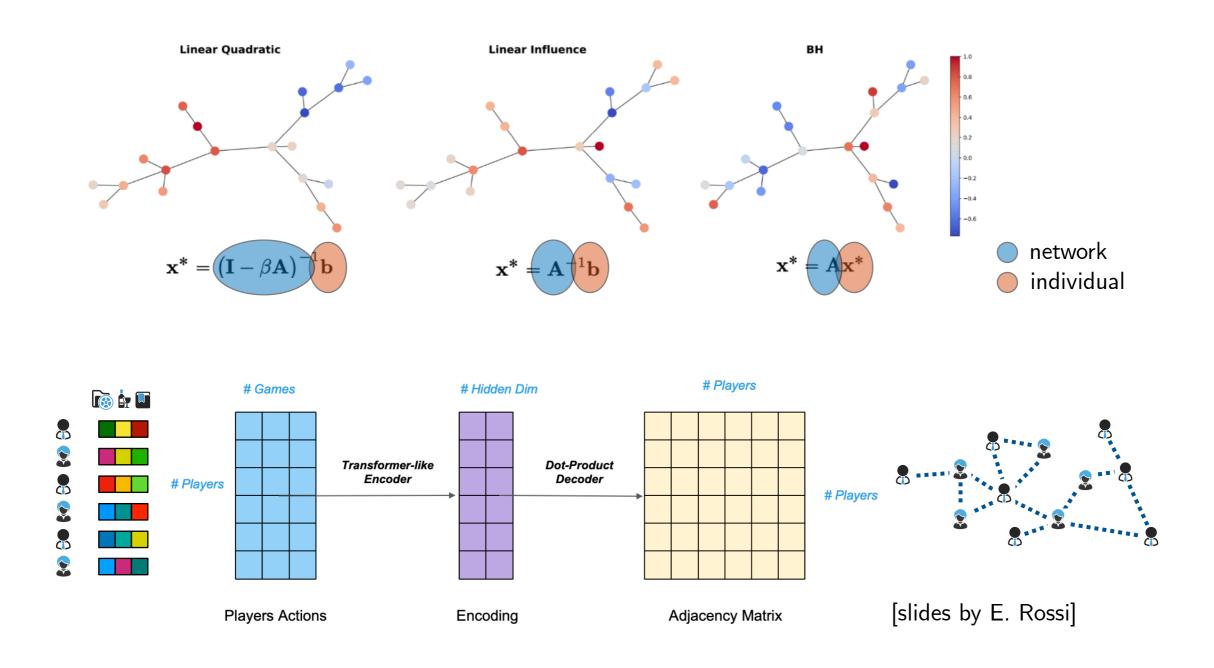


Application V: Learning social interactions

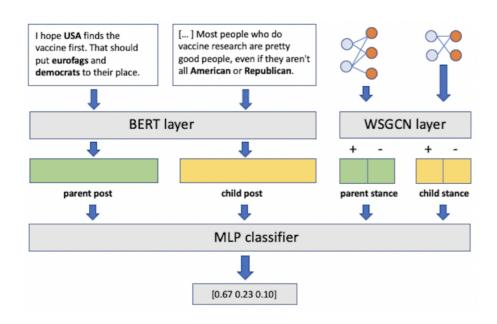




Application V: Learning social interactions

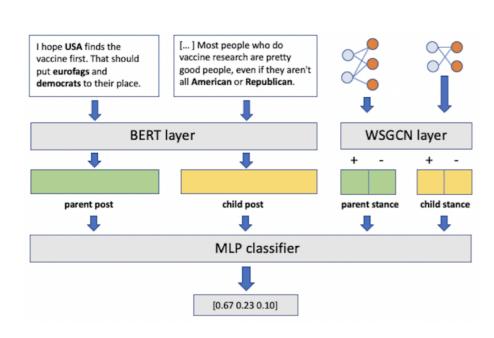


Application VI: Language and social media analysis

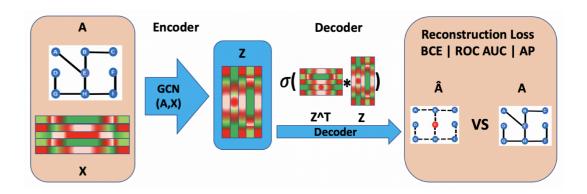


disagreement prediction

Application VI: Language and social media analysis

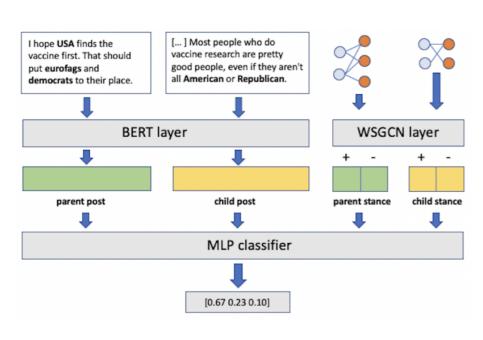


disagreement prediction

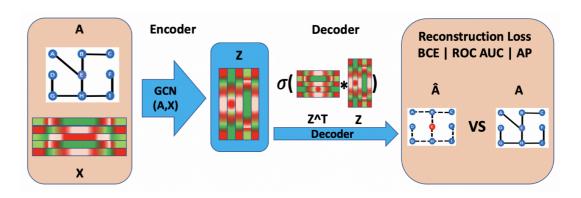


polarisation prediction

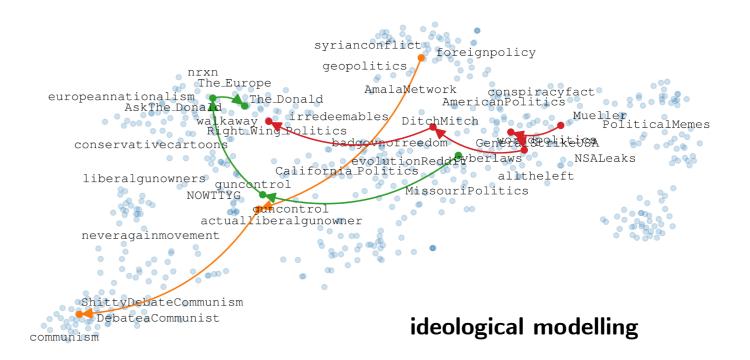
Application VI: Language and social media analysis



disagreement prediction



polarisation prediction



Hofmann et al., "Modeling ideological salience and framing in polarized online groups with graph neural networks and structured sparsity," NAACL, 2022. Zhang et al., "Predicting polarisation of dynamic social networks via graph auto-encoders," IC2S2, 2023.

Some Final Thoughts

Summary

- Graph machine learning
 - fast-growing field that extends data analysis to non-Euclidean domain
 - highly interdisciplinary: machine learning, signal processing, harmonic analysis, applies statistics, differential geometry
- Limitations and open challenges
 - models on directed and signed graphs
 - models for temporal dynamics and online/adaptive settings
 - construction/refinement of initial graphs
 - robustness & generalisation & scalability
 - interpretability & causal inference
 - expect more applications in social sciences & economics!

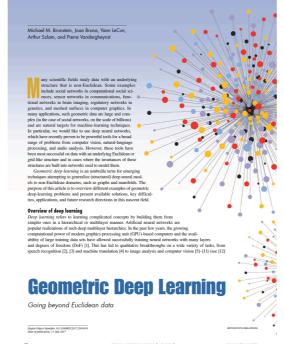
References

David I Shuman, Sunii K. Narang, Pascal Frossard, Antonio Ortega, and Pierre Vandergheynst

The Emerging Field of Signal Processing on Graphs



Extending high-dimensional data analysis to networks and other irregular domains



Neural Networks Zonghan Wu[©], Shirui Pan[©], Member, IEEE, Fengwen Chen, Guodong Long[©] Chengqi Zhang[©], Senior Member, IEEE, and Philip S. Yu, Life Fellow, IEEE

A Comprehensive Survey on Graph

Chengqi Zhang[©]. Senior Member, HEEE, and Philip S. Yu. Life Fellow, IEEE

Abstract—Deep learning has revolutionized many machine learning tasks in recent years, ranging from image desistionities and control of the property of the proper

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Journal of Machine Learning Research 23 (2022) 1-64 Submitted 8/20; Revised 10/21; Published 05/22 Machine Learning on Graphs:

A Model and Comprehensive Taxonomy Ines Chami* Stanford University Stanford, CA, 94305, USA Sami Abu-El-Haija USC Information Sciences Institute Marina Del Rey, CA, 90292, USA Bryan Perozzi
Google Research
New York, NY, 10011, USA Christopher Ré Stanford University Stanford, CA, 94305, USA

Kevin Murphy
Google Research
Mountain View, CA, 94043, USA

Editor: Ruslan Salakhutdinov

Keywords: Network Embedding, Graph Neural Networks, Geometric Deep Learning, Manifold Learning, Relational Learning



Annual Review of Sociology Machine Learning for Sociology

Mario Molina and Filiz Garip

Department of Sociology, Cornell University, Ithaca, New York 14853, USA; email: mm2535@cornell.edu.fazrip@cornell.edu

ANNUAL CONNECT

Abstract

Machine learning is a field at the intersection of statistics and computer science that uses algorithms to extract information and knowledge from data. Its applications increasingly find their way into economics, political science, and sociology. We offer a brief introduction to this vast toolbox and illustrate its current uses in the social sciences, including distilling measures from new data sources, such as set and images, characterizing population beerogeneity; improving causal inference; and offering predictions to aid policy decisions and theory development. We argue that, in addition to serving similar purposes in sociology, machine learning tools can speak to long-standing questions on the limitations of the linear modeling framework, the criteria for evaluating empirical findings, transparency around the context of discovfor evaluating empirical findings, transparency around the context of discovery, and the epistemological core of the discipline.



Annual Review of Political Science

Machine Learning for Social Science: An Agnostic Approach

Justin Grimmer,¹ Margaret E. Roberts,² and Brandon M. Stewart³

¹Department of Policial Science and Hower Institution, Stanford University, Stanford, California 94305, USA; emil: jegrimme@stanford.ehu

**Department of Policial Science and Hacos@b Data Science Institute, University of California Sn Diego, La Jolla, California 92093, USA; emil: meroberss@uscl.edu

**Department of Sociology and Office of Population Research, Princeton University, Princeton, New Jercy, 08740, USA; emil: humbel/princeton.edu

ANNUAL CONNECT

Annu. Rev. Political Sci. 2021. 24:395-419

machine learning, text as data, research design

Abstract

Social scientists are now in an era of data abundance, and machine learning Social sections are from all net as doubtained, and intentine cannies tools are increasingly used to extract meaning from data sets both massive and small. We explain how the inclusion of machine learning in the social sciences requires us to rethink not only applications of machine learning methods but also best practices in the social sciences. In contrast to the tramethods but also best practices in the social sciences. In contrast to the tra-ditional tasks for machine learning in computer science and statistics, when machine learning is applied to social scientific data, it is used to discover new concepts, measure the prevalence of those concepts, assess causal effects, and make predictions. The abundance of data and resources facilitates the move away from a deductive social science to a more sequential, interactive, and ultimately inductive approach to inference. We explain how an agnostic approach to machine learning methods focused on the social science tasks facilitates progress across a wide range of questions.