Learning Quadratic Games on Networks

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(joint work with Yan Leng and Alex ‘Sandy’ Pentland)

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Motivation

Consider a group of students making choices on educational effort
- making effort is costly
- I will benefit from my own effort
- I will also benefit from my friends’ effort
Motivation

• Consider a group of students making choices on buying a book
  - buying a book is costly
  - if a friend of mine will buy, then I won’t buy
  - but if none of my friend will buy, then I will buy
Motivation

- Such strategic interactions can be modelled as games on networks
  - players, actions, payoffs, interaction network
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  - payoff of an individual depends on her action as well as her neighbours’ actions
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  - strategic interactions: complement games or substitute games
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- Economics
  - existence of equilibrium
  - how action/payoff depends on network
  - incentivisation scheme or intervention
  - on a given or predefined network
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- **Economics**
  - existence of equilibrium
  - how action/payoff depends on network
  - incentivisation scheme or intervention
  - on a *given or predefined* network

- **Computer science (graphical games)**
  - algorithms for computing equilibrium
  - *binary* or *finite discrete* action space
Motivation

• Economics
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• Computer science (graphical games)
  - algorithms for computing equilibrium
  - binary or finite discrete action space

• This work
  - learning interaction network given continuous actions in a broad class of games
  - learning marginal benefits
Motivation

• Many examples
  - observe individual decisions (e.g., adoptions), but not social relationship
  - observe R&D activities of firms, but not collaboration networks
  - observe international policies of countries, but not political alliance
Outline

• Background
  - learning network structure from data
  - network games of strategic interactions

• Learning games with linear-quadratic payoffs
  - independent marginal benefits
  - homophilous marginal benefits

• Experimental results

• Discussion
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Learning network structure from data

- Given observations on a number of variables and some prior knowledge (distribution, model, etc)
Learning network structure from data

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- Build/learn a measure of relationship between variables (correlation/covariance, graph topology, or similar)
Learning network structure from data

- Different perspectives in the literature
  - statistical: graph captures **data distribution** (e.g., probabilistic graphical model)

Learning network structure from data

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  Conditional independence

  \[(v_i, v_j) \notin \mathcal{E} \iff x_i \perp x_j \mid x \setminus \{x_i, x_j\}\]

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conditional independence

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probability parameterised by \(\Theta\)

\[
P(x|\Theta) = \frac{1}{Z(\Theta)} \exp\left(\sum_{v_i \in \mathcal{V}} \theta_i x_i^2 + \sum_{(v_i, v_j) \in \mathcal{E}} \theta_{i,j} x_i x_j\right)
\]

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probability parameterised by \( \Theta \)

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P(\mathbf{x} | \Theta) = \frac{1}{Z(\Theta)} \exp \left( \sum_{v_i \in \mathcal{V}} \theta_{i,i} x_i^2 + \sum_{(v_i, v_j) \in \mathcal{E}} \theta_{i,j} x_i x_j \right)
\]

Gaussian Markov random field with precision \( \Theta \)

\[
P(\mathbf{x} | \Theta) = \frac{|\Theta|^{1/2}}{(2\pi)^{N/2}} \exp \left( -\frac{1}{2} \mathbf{x}^T \Theta \mathbf{x} \right)
\]

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  - physics: data correspond to physical process on graph (e.g., network cascade)

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- No game-theoretic aspect of strategic interactions (and no modelling of individual marginal benefits)
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Network games of strategic interactions

- Given a graph $G(V, E)$ with unitary edge weights $G_{ij}$
Network games of strategic interactions

- Given a graph $G(V, E)$ with unitary edge weights $G_{ij}$
- Games with linear-quadratic payoffs

\[
u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in V} G_{ij} a_j
\]
Network games of strategic interactions

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$$u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in \mathcal{V}} G_{ij} a_j$$
Games with linear-quadratic payoffs

\[ u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in V} G_{ij} a_j \]

Remarks
- continuous actions
- for strategic complements (\(\beta > 0\)) and substitutes (\(\beta < 0\))
- can be used to approximate complex non-linear payoffs
- widely adopted in literature [Jackson15,Bramoullé16]

Games with linear-quadratic payoffs

\[ u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in \mathcal{V}} G_{ij} a_j \]

examples
- education: action is educational effort, utility is achievement
- collaboration: action is joint R&D activities, utility is firm profit
- urban dynamics: action is mobility behaviour, utility is social benefit

Games with linear-quadratic payoffs

- Pure-strategy Nash equilibrium (PSNE)

\[ u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in \mathcal{V}} G_{ij} a_j \]
Games with linear-quadratic payoffs

- Pure-strategy Nash equilibrium (PSNE)

\[ u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in \mathcal{N}} G_{ij} a_j \quad \Rightarrow \quad \frac{\partial u_i}{\partial a_i} = b_i - a_i + \beta (G a)_i \]

\[ a = (I - \beta G)^{-1} b \]
Games with linear-quadratic payoffs

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\[ \frac{\partial u_i}{\partial a_i} = b_i - a_i + \beta (G a)_i \]

\[ \mathbf{a} = (\mathbf{I} - \beta \mathbf{G})^{-1} \mathbf{b} \]

**Assumption:** spectral radius of \( \beta \mathbf{G} \) is smaller than 1

- guarantees matrix inversion
- ensures uniqueness and stability of equilibrium action [Ballester06]
Games with linear-quadratic payoffs

- **Pure-strategy Nash equilibrium (PSNE)**

\[ u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in \mathcal{V}} G_{ij} a_j \]

\[ \frac{\partial u_i}{\partial a_i} = b_i - a_i + \beta (Ga)_i \]

\[ a = (I - \beta G)^{-1} b = \sum_{p=0}^{+\infty} \beta^p G^p b \]

- **Assumption:** spectral radius of \( \beta G \) is smaller than 1
  - guarantees matrix inversion
  - ensures uniqueness and stability of equilibrium action [Ballester06]
  - equilibrium related to Katz-Bonacich centrality
  - payoff dependency spreads indirectly through network

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Learning with independent marginal benefits

Nash equilibrium

\[ a = (I - \beta G)^{-1} b \]

\[ (I - \beta G) a = b \]
Learning with independent marginal benefits

Nash equilibrium

\[ a = (I - \beta G)^{-1} b \quad \Rightarrow \quad (I - \beta G) a = b \]

consider \( K \) games

\[ B = [b^{(1)}, b^{(2)}, \ldots, b^{(k)}] \in \mathbb{R}^{N \times K} \quad \text{marginal} \]

\[ A = [a^{(1)}, a^{(2)}, \ldots, a^{(K)}] \in \mathbb{R}^{N \times K} \quad \text{action} \]
Learning with independent marginal benefits

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Joint learning

Minimize
\[ f(G, B) = \| (I - \beta G) A - B \|^2_F + \theta_1 \| G \|^2_F + \theta_2 \| B \|^2_F, \]
subject to
\[ G_{ij} = G_{ji}, \; G_{ii} \geq 0, \; G_{ii} = 0 \quad \text{for} \; \forall i, j \in \mathcal{V}, \]
\[ \| G \|_1 = N, \]
Learning with independent marginal benefits

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Learning with independent marginal benefits

Nash equilibrium

\[ \mathbf{a} = (\mathbf{I} - \beta \mathbf{G})^{-1} \mathbf{b} \quad \Rightarrow \quad (\mathbf{I} - \beta \mathbf{G}) \mathbf{a} = \mathbf{b} \]

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joint learning

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\begin{align*}
\text{minimize}_{\mathbf{G}, \mathbf{B}} & \quad f(\mathbf{G}, \mathbf{B}) = \| (\mathbf{I} - \beta \mathbf{G}) \mathbf{A} - \mathbf{B} \|^2_F + \theta_1 \| \mathbf{G} \|^2_F + \theta_2 \| \mathbf{B} \|^2_F, \\
\text{subject to} & \quad G_{ij} = G_{ji}, \ G_{ij} \geq 0, \ G_{ii} = 0 \ \text{for} \ \forall i, j \in \mathcal{V}, \\
& \quad \| \mathbf{G} \|_1 = N,
\end{align*}
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Learning with independent marginal benefits

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\end{align*}
\]

\[ 19/39 \]
Learning with independent marginal benefits

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$$a = (I - \beta G)^{-1} b \quad \Rightarrow \quad (I - \beta G) a = b$$

consider $K$ games

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joint learning

$$\text{minimize}_{G, B} \quad f(G, B) = ||(I - \beta G)A - B||_F^2 + \theta_1 ||G||_F^2 + \theta_2 ||B||_F^2,$$

subject to

$$G_{ij} = G_{ji}, \quad G_{ii} \geq 0, \quad G_{ii} = 0 \quad \text{for} \quad \forall i, j \in \mathcal{V},$$

$$||G||_1 = N,$$

quadratic programme jointly convex in $G$ and $B$
Learning with independent marginal benefits

Algorithm 1 Learning games with independent marginal benefits

1: **Input** Observed actions $A \in \mathbb{R}^{N \times K}$ for $K$ games, $\beta, \theta_1, \theta_2$
2: **Output** Network $G \in \mathbb{R}^{N \times N}$, marginal benefits $B \in \mathbb{R}^{N \times K}$ for $K$ games
3: Solve for $G$ and $B$ in Eq. (6)
4: **return** $G, B$
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Learning with homophilous marginal benefits

- Phenomenon of homophily in social networks [McPherson01]

Learning with homophilous marginal benefits

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- Given homophily, marginal benefits are smooth functions on the graph

Learning with homophilous marginal benefits

- Phenomenon of homophily in social networks [McPherson01]
- Given homophily, marginal benefits are **smooth** functions on the graph

\[
b : \mathcal{V} \rightarrow \mathbb{R}^N
\]

\[
b^T L b = \frac{1}{2} \sum_{i,j=1}^{N} G_{ij} (b_i - b_j)^2
\]

Learning with homophilous marginal benefits

joint learning

\[
\text{minimize}_{G,B} \quad h(G, B) = \|(I - \beta G)A - B\|_F^2 + \theta_1\|G\|_F^2 + \theta_2\text{tr}(B^T LB),
\]

subject to \quad G_{ij} = G_{ji}, \; G_{ij} \geq 0, \; G_{ii} = 0 \; \forall i, j \in \mathcal{V},

\quad \|G\|_1 = N,

\quad L = \text{diag}(\sum_{j \in \mathcal{V}} G_{ij}) - G
Learning with homophilous marginal benefits

\[ h(G, B) = \|(I - \beta G)A - B\|_F^2 + \theta_1\|G\|_F^2 + \theta_2\text{tr}(B^T LB), \]

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Learning with homophilous marginal benefits

**Joint learning**

Minimize \( h(G, B) = \| (I - \beta G) A - B \|_F^2 + \theta_1 \| G \|_F^2 + \theta_2 \text{tr}(B^T L B), \)

subject to

\[
\begin{align*}
G_{ij} &= G_{ji}, \
G_{ii} &\geq 0, \quad G_{ii} = 0 \text{ for } \forall i, j \in \mathcal{V}, \
\| G \|_1 &= N, \
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\end{align*}
\]
Learning with homophilous marginal benefits

Joint learning

\[
\begin{align*}
\text{minimize} \quad & h(G, B) = \| (I - \beta G) \Lambda - B \|_F^2 + \theta_1 \| G \|_F^2 + \theta_2 \text{tr}(B^T L B), \\
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& \| G \|_1 = N, \\
& L = \text{diag} \left( \sum_{j \in \mathcal{V}} G_{ij} \right) - G
\end{align*}
\]

Not jointly convex in G and B, but convex in subproblems of solving for one while fixing the other.
Learning with homophilous marginal benefits

Algorithm 2 Learning games with homophilous marginal benefits

1: **Input** Observed actions $\mathbf{A} \in \mathbb{R}^{N \times K}$ for $K$ games, $\beta, \theta_1, \theta_2$
2: **Output** Network $\mathbf{G} \in \mathbb{R}^{N \times N}$, marginal benefits $\mathbf{B} \in \mathbb{R}^{N \times K}$ for $K$ games
3: **Initialize** $\mathbf{B}(:, k) \sim \mathcal{N}(0, \mathbf{L}^\dagger)$, $t = 1$, $\Delta = 1$
4: **while** $\Delta \geq 10^{-4}$ and $t \leq \#$ iterations **do**
5: Solve for $\mathbf{G}_t$ in Eq. (9) given $\mathbf{B}_{t-1}$
6: Compute $\mathbf{L}_t$ using $\mathbf{G}_t$
7: $\mathbf{B}_t = (\mathbf{I} + \theta_2 \mathbf{L}_t)^{-1} (\mathbf{I} - \beta \mathbf{G}_t) \mathbf{A}$
8: $\Delta = |h(\mathbf{G}_t, \mathbf{B}_t) - h(\mathbf{G}_{t-1}, \mathbf{B}_{t-1})|$ (for $t > 1$)
9: $t = t + 1$
10: **return** $\mathbf{G} = \mathbf{G}_{\text{iter}}, \mathbf{B} = \mathbf{B}_{\text{iter}}$. 
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Experiments on synthetic data

- Random graphs with 20 nodes
  - Erdős-Rényi (ER): edges created independently with certain probability
  - Watts-Strogatz (WS): regular graph followed by random rewiring
  - Barabási-Albert (BA): graph generated using preferential attachment

- Compute $\beta$ so that $\rho(\beta G) \in (0, 1)$

- Initialise marginal benefits for 50 games
  - independent: $b \sim \mathcal{N}(0, I + \frac{1}{10}I)$
  - homophilous: $b \sim \mathcal{N}(0, L^\dagger + \frac{1}{10}I)$

- Generate equilibrium actions $a = (I - \beta G)^{-1} b$
Setting

- Evaluate on area under the curve (AUC)

- Baselines
  - sample correlation as edge weights
  - graph learned by regularised graphical Lasso [Lake10]

\[
\begin{align*}
\text{maximize} \quad & \log \det \Theta - \text{tr} \left( \frac{1}{M} XX^T \Theta \right) - \rho \| \Theta \|_1, \\
\text{subject to} \quad & \Theta = L + \frac{1}{\sigma^2} I, \quad L \in \mathcal{L},
\end{align*}
\]

Learning interaction network

independent marginal benefits
Learning interaction network

independent marginal benefits

homophilous marginal benefits
Performance vs. regularisation parameters

- Homophilous marginal benefits

\[
\begin{align*}
\text{minimize} \quad & h(G, B) = \| (I - \beta G) A - B \|_F^2 + \theta_1 \| G \|_F^2 + \theta_2 \text{tr}(B^T L B)
\end{align*}
\]
Performance vs. number of games

- Homophilous marginal benefits with $\rho(\beta G) = 0.6$
Performance vs. noise intensity

- Homophilous marginal benefits with $\rho(\beta G) = 0.6$
Performance vs. network structure

- Homophilous marginal benefits with $\rho(\beta G) = 0.6$

- Parameters in random graph models
  - ER: each node pair connected with probability $p$
  - WS: $k$-regular graph with rewiring probability $p$
  - BA: $m$ nodes added at each graph generation step
Performance vs. strength of homophily

- Homophilous marginal benefits with $\rho(\beta G) = 0.6$
- Marginal benefits $B$ as linear combinations of 1st-5th (strong homophily), 6th-10th (medium), 11th-15th (weak) eigenvectors of graph Laplacian
Learning marginal benefits

- Compare groundtruth and learned marginal benefits using coefficients of determination ($R^2$)

Table 1: Performance (in terms of $R^2$) of learning marginal benefits.

<table>
<thead>
<tr>
<th></th>
<th>Algorithm 1</th>
<th>Algorithm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>ER graph</td>
<td>0.959</td>
<td>0.005</td>
</tr>
<tr>
<td>WS graph</td>
<td>0.955</td>
<td>0.007</td>
</tr>
<tr>
<td>BA graph</td>
<td>0.937</td>
<td>0.008</td>
</tr>
</tbody>
</table>
Experiments on real world data

• Learning social network
  - 182 households in a village in rural India [Banerjee13]
  - actions in 31 games: number of facilities adopted by each household
  - strategic complements: conformity to social norms (decisions made by neighbours)
  - compare with ground truth self-reported friendship

Experiments on real world data

• Learning social network
  - 182 households in a village in rural India [Banerjee13]
  - actions in 31 games: number of facilities adopted by each household
  - strategic complements: conformity to social norms (decisions made by neighbours)
  - compare with groundtruth self-reported friendship

• Learning trade relationship
  - 235 countries
  - actions in 192 games: import/export of 96 products of countries in 2008
  - strategic substitutes: more demand leads to less utility trading with high-demand countries (same applies to supply)
  - compare with groundtruth: total trades between each pair of countries in 2002

Experiments on real world data

- Performance in learning interaction networks

Table 2: Performance (in terms of AUC) of learning the structure of the social network and the trade network.

<table>
<thead>
<tr>
<th></th>
<th>Social network</th>
<th>Trade network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample correlation</td>
<td>0.525</td>
<td>0.523</td>
</tr>
<tr>
<td>Regularized graphical Lasso</td>
<td>0.564</td>
<td>0.570</td>
</tr>
<tr>
<td>Algorithm 1</td>
<td>0.575</td>
<td>0.622</td>
</tr>
<tr>
<td>Algorithm 2</td>
<td>0.576</td>
<td>0.677</td>
</tr>
</tbody>
</table>
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- Learning political network
  - 26 cantons in Switzerland
  - actions in 37 games: percentage of supportive votes in referendums in 2008-2012
  - strategic complements: political alliance
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- Learning political network
  - 26 cantons in Switzerland
  - actions in 37 games: percentage of supportive votes in referendums in 2008-2012
  - strategic complements: political alliance
  - apply the spectral clustering algorithm [Ng02] to the inferred network

Outline

• Background
  - learning network structure from data
  - network games of strategic interactions

• Learning games with linear-quadratic payoffs
  - independent marginal benefits
  - homophilous marginal benefits

• Experimental results

• Discussion
Discussion

- Applications in practical scenarios
  - detect communities of players (for stratification)
  - compute centrality measures (for efficient targeting strategies)
  - design intervention mechanisms to achieve planning objective
    - maximise total utilities of players (via adjusting marginal benefits) [Galeotti17]
    - reduce inequality between players (via adjusting interaction network)
Discussion

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• Open issues & future directions
  - determination of $\beta$ (strength of strategic interaction)
  - probabilistic interpretation of learning framework
  - more general payoff functions
  - partial/incomplete observations
  - dynamic interaction networks