Creating Aperture Masks in Phase Space

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Abstract: The Wigner distribution is used to model the PSF of an aperture mask at different defocus planes. Algorithmic methods of determining an optimal mask pattern for a desired set of impulse responses are investigated.

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1. Introduction

Historically, the goal of most camera designs was to minimize the size of the point spread function (PSF) over an extended depth range, allowing a sharp image to fall onto a permanent recording medium like film. With the advent of digital sensor technology, direct access to the intensity information of light for computation became possible. Image processing techniques like deconvolution, edge enhancement, or feature detection are increasingly common options in simple consumer cameras. Thus, the purpose of a lens to minimize the size and shape of a PSF is no longer completely vital, when blur from the PSF can be removed computationally.

Instead, there have been several recent attempts to manipulate the PSF of a camera to gain additional information about a scene. Aperture phase or amplitude masks can yield useful depth information [1, 2], or extend a camera's depth-of-field [3-5]. This paper attempts to build a general framework for the creation of aperture masks that can respond to defocus in an arbitrary fashion.

The foundation of this framework will rest in phase space, where both the spatial and frequency content of light can be evaluated. Under geometric optics assumptions, light propagation is easily modeled using the position and direction of a particular ray using a series of square matrices (i.e., the ABCD-matrix formalism). The Wigner distribution is a convenient space-spatial frequency phase space representation that operates in the physical optics realm [6]. While primarily used with coherent optics systems, it has also been extended to describe incoherent imaging setups [7, 8]. Through a few direct transformations, the Wigner distribution can directly output the PSF of a camera at a given degree of defocus. We will use these simple properties to approach the inverse problem of finding an optimal aperture mask for a desired set of PSFs.

2. Wigner Distribution as a Design Tool

The Wigner distribution W simultaneously presents spatial (x) and local spatial frequency (u) content of a monochromatic system in a single plot. It is given as the Fourier transform of the autocorrelation of a certain wavefront. If a transparency is illuminated by a coherent plane wave parallel to the z-axis, then the resulting Wigner distribution after passing through the transparency is simply the Wigner of the transmission mask function:

$$W_{lens}(x,u) = \int t\left(x + \frac{x'}{2}\right) t^*\left(x - \frac{x'}{2}\right) e^{-2\pi i x' u} du. \tag{1}$$

Considering an imaging setup, the propagation of this wavefront from the aperture to the sensor is described with a Fourier transform, which is given as a 90° rotation of the Wigner distribution,

$$W_{sensor}(x,u) = W_{lens}\left(-u\lambda f, \frac{x}{\lambda f}\right), \tag{2}$$

where the x and u coordinates have been traded. As with a geometrical light field [8], free-space propagation a distance d along the z-direction is a shear transformation:

$$W_{misfocus}(x,u) = W_{sensor}(x - \lambda d u, u). \tag{3}$$

Finally, the intensity distribution of the wavefront can be found as a projection (integration) along u. Through these three simple operations, the PSF response of a mask t can be modeled at different amounts of misfocus directly.

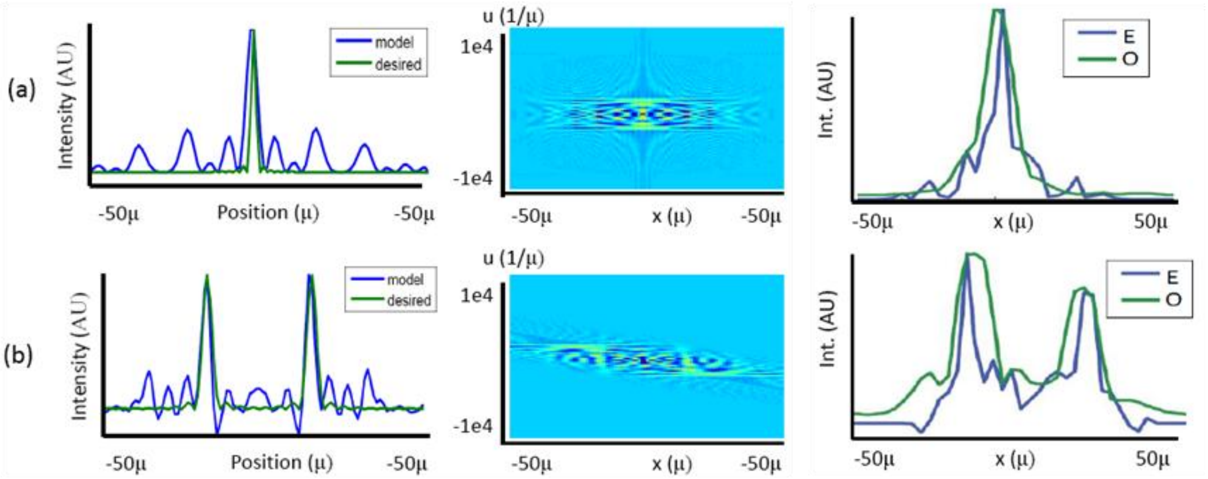

Figure 1: An example global search output for a 1cm symmetric mask with 50 binary elements using an f/4 lens. (a) In-focus, a centered sinc is desired. (b) At .01mm misfocus, two offset sinc patterns are desired. (left) These two desired PSFs are shown in plots in green, with the modeled intensity from the optimal mask in blue. (middle) The corresponding Wigner distributions, which generate the best-fit intensity upon integration along u. (right) Experimental results for this aperture mask. The trace through observed results (O) compared with expected (E) for an in-focus and .01mm defocused system.

3. Global Search Solution

Determining an aperture mask from a desired set of PSFs is an easily approachable problem under this framework. The most direct solution is a global search of mask functions. Starting with an initial mask estimate and applying (1)-(3) for different distances of sensor defocus d, mask performance can be compared to a desired intensity distribution at these defocus settings using mean-squared error as a performance metric. Iterating through a large set of discrete mask functions is possible because of the computational simplicity of operations. For example, a search over the 2^{25} symmetric 1D binary amplitude mask patterns of 50 discrete pixels is presented in Figure 1.

4. Mask Inversion Algorithm

A direct inversion procedure can be designed using a phase space defocus model instead of, for example, the Fresnel propagation equation. The inversion problem can be simplified into two steps. The first step is to create a Wigner distribution from a desired set of PSFs. Since each PSF is a projection of the same Wigner distribution at a slightly different angle (shear), this procedure resembles a common tomographic imaging problem.

This becomes clear when working with the 2D Fourier transform of the Wigner distribution, the ambiguity function. It is well known that slices through the origin of the ambiguity function at different angles produce the misfocused optical transfer functions (OTF) of an imaging system [7]. Thus, each desired PSF can be transferred to a corresponding OTF, which can begin to populate an initial ambiguity function estimate. This estimate can be filled in using a linear algebra tool like a matrix completion or matrix recovery algorithm, which work well with sparse data sets. The Wigner distribution can be represented as a sparse matrix, as discussed in the next section.

The second step to create an aperture mask is to invert this Wigner distribution guess into a mask. From (1), *t(x)* can be found up to a constant phase factor [9]:

$$t(x_1)t^*(0) = \int W_{lens}\left(\frac{x_1}{2}, u\right) e^{2\pi i x_1 u} du. \tag{4}$$

Going from 2D to 1D, it is clear that the Wigner distribution is a highly redundant representation of an imaging system. Thus, care must be taken in choosing desired PSFs for input.

5. Space of Wavefront Distributions

It is well known that a mask function $t(x_1)$ cannot create an arbitrary set of intensity PSF patterns at different focal planes, but (4) indicates why. The imaging system's Wigner distribution, which completely defines its performance,

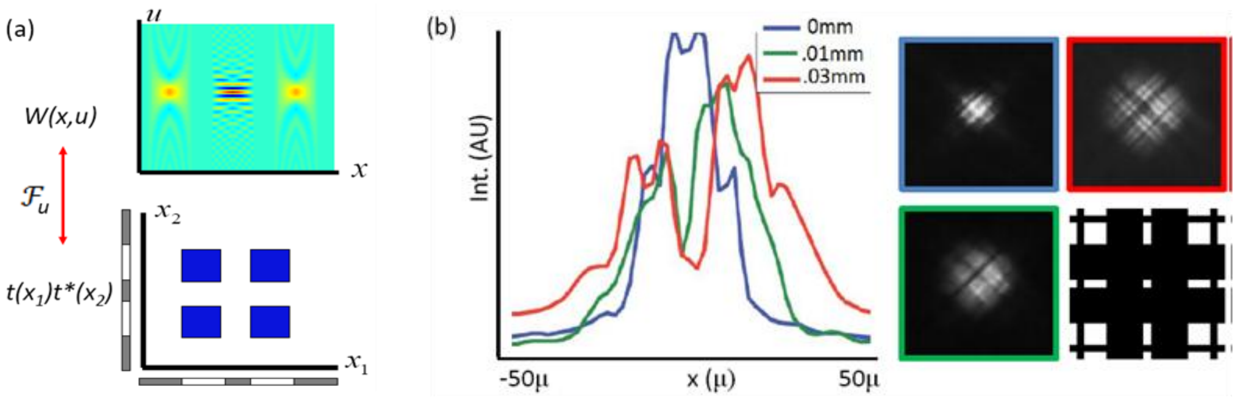

Figure 2: (a) Diagram showing the Wigner distribution and mutual intensity of a two-slit mask. They are related through the inverse Fourier transform in (4), and the mutual intensity is rank-1. (b) Experimental plot and images for an aperture mask using 3 desired PSFs as input: one sinc function in focus (blue), two at .01mm misfocus (green), and four sinc functions at .03mm misfocus (red). The optimal mask pattern is shown in the lower right. The limited amount of intensity variation becomes clear using three intensity inputs to optimize a 2D function that can be expressed as a rank-1 matrix.

can be reduced to a rank-1 matrix in mutual intensity space. This is a result of using an assumption of spatially coherent light, which is a requirement of PSF measurement. In other words, since multiple 1D projections create a 2D Wigner distribution estimate, and (4) tells us that this distribution can be expressed as a 1D function in some space, it is limited in flexibility. An optimal mask pattern will almost always yield approximations to a desired set of PSFs.

It is interesting to note that the rank of the Wigner in mutual intensity space increases with partially coherent illumination, until it is a full-rank matrix using incoherent illumination. A similar rank-1 condition can be found using two amplitude modulation planes under incoherent light assumptions instead of one. For example, the 2D light field of a parallax barrier display can always be decomposed into an outer product of two 1D vectors.

6. Experimental Results

The performance of two different aperture masks, designed using the global search algorithm in section 3, is presented in Fig. 1(right) and Figure 2(b). Each 1cm^2 binary amplitude mask is printed at 25µ resolution and inserted above the aperture stop of a Nikon f/1.8 50mm lens mounted on a 5µ pixel Canon DSI SLR. The first mask tested is optimized to the 2 PSF intensity distributions in Figure 1(left), and achieves a close approximation. The second mask tested is optimized to 3 desired PSFs. It is clear that performance becomes limited as the number of intensity patterns used as input increases. This is connected to the rank-1 limitation of all Wigner distributions.

7. References

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