

# Alternative Models of the Rotating Beam

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**Abstract:** Rotating beams are usually analyzed under a Gauss-Laguerre modal decomposition. We examine rotating beam generation and propagation from a modeling perspective, using phase-retrieval algorithms and Hamiltonian ray-tracing, adding flexibility and insight to the design process.

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## 1. Introduction

Controlling the intensity behavior of a wavefront along its axis of propagation can assist, for example, with optical manipulation, trapping, and vortex generation. A particularly interesting beam with applications in three-dimensional imaging is the rotating beam [1, 2]. A certain class of diffracting elements placed in the aperture plane of a digital camera can yield a point-spread function that rotates with defocus, providing a direct indication of object depth. Piestun et al. have found numerous applications for this element, for example, in a camera and a microscope [3, 4].

Typically, a rotating beam is generated from considering a Gauss-Laguerre (GL) modal beam decomposition, which offers stable solutions to the scalar Helmholtz equation with rotational symmetry about the propagation axis. A superposition of two or more GL modes offers one solution to a propagation-intensity eigenfunction equation, generating intensities that repeat along the propagation trajectory [5]. The mathematical theory behind these coherent modal superpositions is well understood.

We approach the problem of generating and modeling the rotating beam from several new perspectives. First, we show how approximate forms of rotating beams can be created through the use of iterative phase retrieval procedures, with two different algorithms. Using a few desired intensities as input, these algorithms converge to a solution for an amplitude and phase distribution to approximate a three-dimensional intensity distribution that rotates along the propagation axis. New forms of rotating beams, like partially coherent beams, can be explored using a non-linear optimization approach. Second, the propagation of rotating beams within a GRIN medium is examined. Rotation rate and beam size oscillate as a function of propagation distance, offering a unique space to examine beam angular momentum.

## 2. Generation of Rotating Beams using Phase Retrieval

Phase retrieval algorithms solve for a wavefront's amplitude and phase using intensity distributions at multiple planes along the propagation axis as input. A simple rotating beam estimate can be designed, for example, with a two-spot intensity distribution at three different rotation angles as input (Fig. 1(a)). In this example, we solve for the optimal amplitude and phase shown at the right of Fig. 1 using a phase-retrieval algorithm developed by Fienup [6], but based on Fresnel propagation [7]. The beam achieves an average MSE of .0029 from three desired intensity distributions after 35 iterations for the rotating 2-point example (Fig. 1(b)), and an MSE of .0031 for the rotating 4-point example (Fig. 1(c)). The intensities are defined with a constant rotation angle 0.2mm and 0.6mm away from the Fourier plane, as the rotation rate follows a quadratic relationship with propagation distance [5].

More importantly, the derived amplitude and phase distribution produces a beam that continues to rotate well beyond the furthest desired intensity plane, as shown in green boxes in Fig. 1, which are recorded at a plane twice as far as the last desired intensity (1.2mm away). Thus, the algorithm converges to an amplitude and phase distribution that adheres to a bound state of rotation within a limited window of propagation. A large variety of parameters can be directly examined simply by altering the three inputs with phase retrieval, including point separation, spot width, rotation rate, and even a variable rotation rate, which will also be explored when the rotating beam is analyzed within a GRIN medium. Furthermore, working within a phase retrieval framework allows for constraints to be placed directly on the amplitude and phase transmittance of the diffracting mask.

A second possible method of modeling a desired intensity distribution in 3D, termed mode-selective [8], is based on the ambiguity function (AF). This algorithm iteratively converges to an optimal AF to approximate a desired set

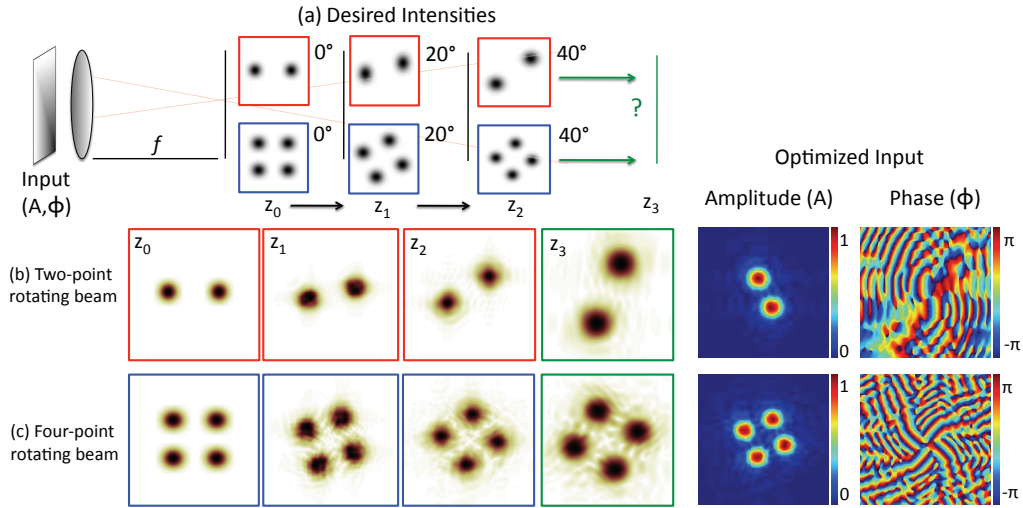


Fig. 1: (a) The phase-retrieval algorithm in [7] is used to find an input amplitude and phase aperture mask to create the desired 3 intensity distributions for a two-point rotating beam (red) and, separately, a 4-point rotating beam (blue), which each rotate to  $40^\circ$ . After 35 iterations, the algorithm converges to the amplitude and phase shown at right, which produces the 4 intensities shown to the left for the two-point (b) and four-point (c) rotating beams. The green-boxed intensity pattern is determined at  $z_3 = 2z_2$  (i.e., beyond the region of intensities for algorithm input), demonstrating convergence to a beam that continues to rotate with propagation.

of intensities at different propagation planes. Unlike phase retrieval, the rank-selective model offers a direct way to optimize over an arbitrary degree of partial coherence. While computationally more intensive, examining partial coherence may allow for the generation of a larger variety of rotating beam 3D intensity patterns. In an imaging setup, a partially coherent rotating point spread function can be simulated using a dynamic (instead of fixed-pattern) mask.

### 3. Propagation of Rotating Beams in GRIN Media

Extending the analysis of rotating beam propagation into complex media with an index of refraction  $n(x, y, z)$  that varies with position requires the use of Hamiltonian ray-tracing. However, rotating beams are fundamentally based upon the physical-optic effects of diffraction and interference, and are rarely treated within a ray framework. The Wigner distribution (WDF) is one particular phase space model that offers a direct connection between physical optics and ray-tracing [9].

Specifically, the WDF's 4-dimensional representation of the spatial  $(x, y)$  and spatial frequency content  $(u, v)$  of a wavefront can equivalently be treated as a set of weighted ray positions and angles, otherwise known as a geometric light field [10]. Each ray  $r(x_i, y_i, u_i, v_i, z = 0)$  in this weighted set can be input into a Hamiltonian solver for a particular  $n(x, y, z)$ , which will trace the ray to a final position and angle at a later plane,  $r(x_f, y_f, u_f, v_f, z = z_f)$ , where it's value

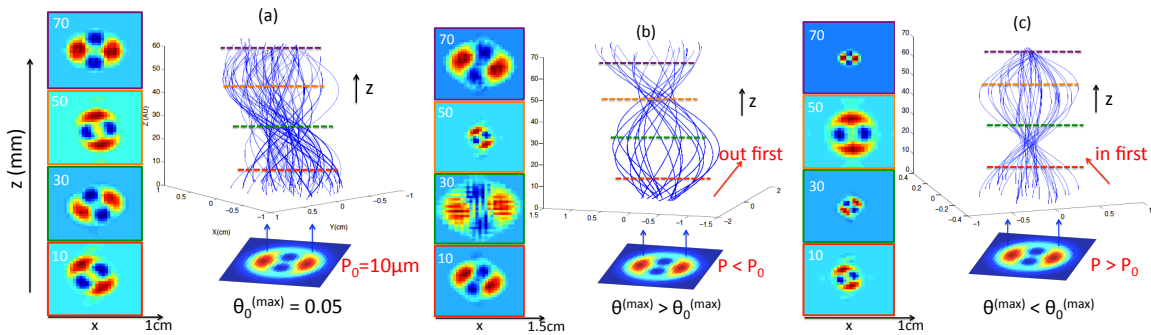


Fig. 2: The WDF of a two-point rotating beam can be coupled with Hamiltonian ray-tracing to examine the rotating beam's intensity as it propagates through an elliptical GRIN. Depending upon the grating's initial feature size  $p$ , the beam will either (a) maintain a constant speed and size ( $p = 10\mu\text{m}$ ), (b) oscillate by initially shrinking ( $p = 4\mu\text{m}$ ) or (c) oscillate by initially growing ( $p = 20\mu\text{m}$ )

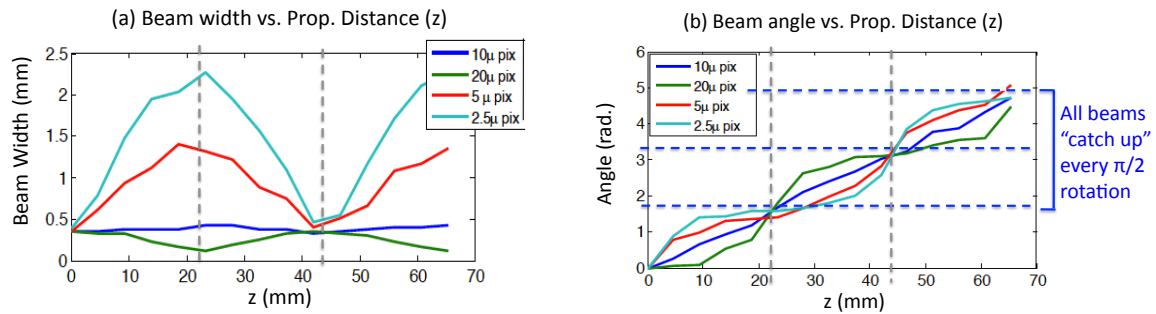


Fig. 3: (a) Rotating beam size oscillates with distance for the beams in Fig. 2 (b) The beams also rotate at non-uniform speeds, but align to the same orientation (but not size) every quarter turn. Beam width and angle are estimated from modeled data.

will be added to a total 2D intensity measurement at the plane  $z_f$ .

This WDF-ray tracing model is implemented within an elliptical GRIN fiber with an index distribution  $n(x, y, z) = \sqrt{1 - k^2(x^2 + y^2)}$  (Fig. 2). Experimentally, this model can be realized with the diffracting amplitude and phase distribution on the right of Fig. 2(b) placed directly against a GRIN fiber and illuminated with a plane wave. Depending upon the initial diffraction angle into the GRIN medium, varying oscillations in beam size and rotation speed can be observed. The degree of initial diffraction is set by the minimum feature size of the diffracting surface that creates the rotating beam. For the beams in Fig. 2, we modeled a rotating beam as a superposition of  $GL(0, 0)$  and  $GL(2, 2)$  modes with a WDF of  $32^4$  resolution into a 70mm long elliptical GRIN with  $k = .15$  and  $\lambda = .5\mu\text{m}$ .

The size of the aperture  $w$ , and hence minimum feature, is varied from Fig. 2(a)–(c), which changes how the beam size and rotation speed oscillate. The beam in Fig. 2(a) maintains a constant width and rotation speed with  $w = .65\text{mm}$ . Decreasing  $w$  causes the beam to initially decrease in size but speed up in rotation (b), while increasing  $w$  does the exact opposite (c). This fluctuation in beam size and rotation angle is plotted for 4 different values of  $w$  in Fig. 3. This inverse relationship is conceptually similar to the conservation of angular momentum in classical mechanics. Although rotating beams with varying maximum diffraction angles also vary in rotational speed, they align every quarter rotation at the same locations (here at  $z=22\text{mm}$  and  $44\text{mm}$ ). These locations correspond to the locations of maximum/minimum beam width, and are also the locations of the GRIN fiber's caustics.

#### 4. Conclusion

We present a framework to generate and alter rotating beams based upon non-linear optimization strategies, in both free space as well as varying index media. Future work will focus both on generating partially coherent rotating beams, as well as the inverse problem of beam generation within a GRIN media.

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