Towards a Common Framework for Parallax Barrier and Holographic 3D Display

Abstract
This paper connects parallax barriers, which are typically modeled using geometric optics, and holograms, which are often considered assuming light is a wave. While both displays generate three-dimensional imagery, the function and tradeoffs of each are rarely directly compared. We connect the two approaches under a general framework based in phase space, from which insights into applications and limitations of both parallax-based and holography-based systems can be observed. We show that each display form generates a light distribution that can always be expressed as an algebraic rank-1 matrix. We then use this rank-1 limitation to propose a new form of hybrid 3D display, which takes advantage of both geometric and physical optic–effects to achieve a more flexible display output.

1 Introduction
Parallax barrier 3D displays are conventionally treated under a ray-based framework, and thus receive considerable attention from the graphics community. Holographic displays, on the other hand, are typically explained using physical–optics concepts, which may deter those more familiar with ray–tracing and rendering from exploring their benefits. Based on recent connections made between geometric and physical optics by [Zhang and Levoy 2009], this paper attempts to describe holography in a format amenable to a graphics audience. Encouraged by present scaling trends in display pixel size, we summarize the limitations of 3D displays within a linear algebraic framework, aiming to partner holography with parallax barrier display for future display design.

In their simplest form, parallax barriers are comprised of a plane of pixels and a plane of light-modulating slits (Figure 2(a)). The pixels contain a mix of spatial and angular content in the form of multiple interlaced images, and the slits direct rays from each image to different viewing locations. Properly calibrated, a parallax barrier can deliver a unique image to each eye of a viewer, yielding a 3D image without the need of special glasses.

Holograms also contain a mixture of spatial and angular content, but in the form of interference fringes. A good overview of their basic form can be found in [Goodman 1996]. Recent work in holography has focused on generating these fringes computationally, which this paper will concentrate on. A computed holographic fringe pattern can be displayed as a grayscale 2D image on a high-resolution screen. When illuminated with coherent light source, like a laser, it produces a 3D image of the encoded content. Physically, the grayscale holographic screen is much like the screen of a parallax barrier, but at resolution scales closer to the wavelength of visible light (i.e., pixels on the order of a few μm instead of 100’s of μm). Current displays are approaching 100 μm pixels [Optics.org 2008]. To fully develop the connection between parallax barrier and holographic 3D display, this paper provides the following three key analyses:

Contributions
1. A side-by-side performance evaluation of parallax barrier and holographic displays based upon discretizing spatial and angular resolution, which will hopefully clarify holographic operation to the unfamiliar. This comparison includes an experimental verification.
2. A novel linear algebra analysis, based in phase space, which demonstrates the limited space of incoherent and coherent 3D

Figure 1: Parallax barriers and holograms are two forms of 3D display that are typically considered under different physical interpretations, but actually consist of similar underlying phenomena. We will discuss their similarities, and show that both are constrained in what distributions of light they can display. (a) A parallax barrier, comprised of two screens, can only create an algebraic rank-1 incoherent light field. (b) The light field of a hologram can also be reduced to a rank-1 representation. (c) Merging the two display forms together increases the light field’s algebraic rank in both spaces. We provide a framework that begins to encompass both the geometric effects of a parallax barrier and the diffractive effects of a hologram, which will encourage work at their intersection.

Figure 2: (a) A parallax barrier (a screen of pixels covered by a series of slits) can easily create a specific ray, but can only create a discrete approximation of a curved wavefront. (b) An amplitude hologram (one screen of wavelength-scale pixels) can create the same ray with a sinusoidal pattern, but will unavoidably generate two additional rays. It can easily create a curved wavefront with a narrow opening.
light distributions that parallax barriers and holograms can create. We will show that this space is always rank-1 in a certain domain, indicating directions for future research in alternative 3D displays to enhance performance.

3. A new hybrid 3D display format that combines a holographic diffractive screen with a second modulation layer, similar to that of a parallax barrier. This hybrid display generates a set of completely uncorrelated 4D light fields depending upon the direction of illumination, offering a 6D display format [Fuchs et al. 2008] using just two modulation screens.

2 Related Work

Basic Holography The first genre of holograms was a film-based media [Gabor 1948; Leith and Upatnieks 1965], having different properties if thick or thin, or phase or amplitude [Urbach and Meier 1969]. As noted earlier, this paper will be primarily concerned with holographic patterns that can be generated on a computer, some of the first of which were presented by Lohmann [1967]. Since these early efforts, computer-generated holograms (CGH) have been applied widely to 3D display ([Benton and Bove 2006]). We will approach CGH’s using a unique linear-algebraic model inspired by [Ozaktas et al. 2002], which will clarify display limitations.

Advanced Holography CGH’s have been extended to model intensity distributions at multiple planes along the direction of propagation [Dorsch et al. 1994] and into a continuous volume [Piestun et al. 1996]. Spatially–multiplexed holograms, like the holographic stereogram [McCrickard and George 1968], display many discrete viewpoints of an object similar to a parallax barrier display. They often exhibit parallax only along the horizontal direction and provide a user with the appearance of a fully 3D object [Halle 1994], similar to holographic displays using “hogels” [Lucente and Galayen 1995]. Holograms can also be multiplexed over time [Hihaire et al. 1991]. A helpful introduction to various holographic forms is in [Plesniak and Halle 2005]. We propose a mathematical foundation for these varied holographic multiplexing methods, and extend it to offer a diffractive display using two modulation planes, inspired by [Barlet 1984].

Parallax Barriers The first parallax barrier setup, developed by Ives [1903], was a binocular display delivering two slightly different image perspectives to each eye. In-depth comparisons between lenticular, barrier and similarly related integral imaging systems can be found in [Okoshi 1976] and [Javidi 2009], and a physical optics perspective of these devices is in [Moller and Travis 2005]. We will treat all of these two-plane systems in a simplified manner under a light field–based framework similar to Zwicker et. al [2006], offering insights into their relationship with holograms.

Comparisons Recent work in graphics also merges wave-based and geometric light propagation [Zhang and Levoy 2009; Ziegler et al. 2007], which our model will build upon. Furthermore, simple comparisons between geometric-based and holographic displays have been performed in the past [Halle 1997; Halle 1994], as have attempts at integrating display forms from the two genres [Bimber 2004; Plesniak and Halle 2005]. Previous work has not considered their connection from an algebraic-rank viewpoint, which is our focus. Our analysis will use phase space functions like the light field [Levy and Hanrahan 1996] and the Wigner distribution, which is used in physical optics [Bastiaans 1979; Testorf et al. 2009]. Holograms have been analyzed from these phase space perspectives in [Testorf and Lohmann 2008], but were not directly compared to parallax barriers, as we will do.

3 A Direction Comparison

Three key differences between the display media should be kept in mind throughout the following simple comparison. First, parallax barriers operate in the geometric optics limit, while holograms require the physical optics principle of diffraction, due to the scale of the display pixel being close to the wavelength of light. A connection between geometric and physical optics is created with a phase space model, presented in Section 4. Second, parallax barriers operate with incoherent light, while the holograms considered here rely upon a coherent light beam, like such as a laser, which will be discussed in Section 5. Third, a parallax barrier display presents one 3D virtual image to a viewer, while the holograms we will consider create both a real and virtual image. We will focus on the hologram’s virtual image, since it is conceptually equivalent to a parallax barrier’s image. The real image is considered in detail in the supplementary material.

3.1 Parallax Barrier Operation

A generic 1D parallax barrier configuration contains two planes, $s_1$ and $m_2$, with no additional optical elements (Figure 4(a)). In this section, we will assume light from each pixel in $s_1$ only travels through one slit in $m_2$, and that it does not diffract. For a parallax display with $N_p$ pixels in $s_1$, it is clear that there is a direct tradeoff between the amount of spatial and angular content that can be directed to an optimal viewing plane $v_p$. Specifically, angular resolution $\theta_p$ can be given by the number of pixels under each slit, and spatial resolution $x_p$ as the total number of slits (Figure 5(a)).

The total number of pixels in $s_1$ ($N_p$) is thus a combination of this spatial and angular content:

$$x_p \theta_p = N_p. \tag{1}$$

At the optimal viewing position, one ray from each slit will enter one eye, and a discrete number of $\theta_p$ views are visible from different positions along $v_p$. Discretization is one of the main drawbacks of a parallax barrier, and leads to issues like aliasing and pseudoscopic views. High angular resolution is desirable to create a more seamless viewing experience, but parallax barrier displays scale poorly with an increase in resolution for a fixed size $h_p$. As angular resolution increases, $m_2$ decreases in light efficiency for a fixed pixel size, since the optimal slit width in $m_2$ ($r_p$) is equal to the width of one display pixel.
in s1 [Okoshi 1976]. Lenticular arrays can be used instead of slits in s2 to improve optical efficiency, but they will not completely overcome the second issue of diffraction. A slit of width \( r_p \) will diffract a ray across an angle given by

\[
\sin \alpha_p = \frac{\lambda}{2 r_p},
\]

where \( \lambda \) is the light’s wavelength. Thus, as geometric optic-based systems scale towards smaller pixel and barrier widths (e.g., commercially available color screens with 11\(\mu\)m pixels [Optics.org 2008]), physical optics effects cannot be ignored. Table 1 summarizes many of the main properties of this simplified parallax barrier with its physical optic-based counterpart, the hologram.

### 3.2 Simplified Hologram Operation

Diffraction is exactly how a hologram achieves image creation. A “conventional” thin, amplitude-only transmission hologram creates a single ray (i.e., a beam of finite width) by replacing the parallax barrier’s single pixel and slit with a small sinusoidal grating and a barrier which blocks two of the three diffraction orders (Figure 3). The finite width of the ray a sinusoidal grating creates through diffraction is given by,

\[
\Delta \alpha_H = \frac{\lambda z_H}{l_H},
\]

where \( z_H \) is the image distance and \( l_H \) is the grating period. Comparing Eq. 3 to Eq. 2, we see the hologram’s image sharpness improves with a smaller display pixel (\( l_H \)), while a parallax barrier’s sharpness decreases. A basic Fourier hologram, which creates one real 2D image from one perspective, is a summation of these sinusoidal gratings that diffract light into different viewing directions. This single 2D image is proportional to the Fourier transform of the screen pattern, and is conceptually similar to the 2D image of a parallax barrier display seen from one perspective.

Turning this 2D image into a 3D image requires that we convert the Fourier hologram into a Fresnel hologram. Upon coherent illumination, a Fresnel hologram creates a virtual image in the hologram’s “near-zone”, discussed in detail in [Goodman 1996]. Unlike a parallax barrier, this image offers continuous angular content and full depth cues, but suffers from speckle noise and the lack of multiple colors common to most forms of display utilizing a single coherent light source.

### 3.3 Hologram Discretization

Although not exact, a convenient way to construct a Fresnel hologram is simply by tiling together many Fourier holograms. Hologram discretization is used by more advanced holographic forms like the rainbow hologram [Benton 1969], holographic stereograms [McCricker and George 1968], and Lucente’s "holog" based designs [1994; 1995], which all spatially multiplex the holographic screen in different fashions. For simplicity, the terms “spatial multiplexing” and “discretization” will be used interchangeably. Dividing a Fresnel hologram into discrete "patches" is similar the division of the parallax barrier into spatial and angular resolution components (Figure 5(b)). Each Fourier hologram patch is the scaled Fourier transform of the 3D image from one unique perspective (i.e., from one viewing angle). Geometrically, rays can be traced to and from each independent Fourier patch at angles dependent upon their composition of sinusoids, much like rays from a parallax barrier slit are traced at angles dependent upon the \( \phi_p \) pixels beneath it, which greatly aids in CGH design efficiency [Lucente 1994].

The number of required pixels to fully reconstruct the entire parallax content of an image of height \( l_{m1} \) using a Fresnel hologram of height \( h_H \) is

\[
N_H = h_H (h_H + l_{m1}) / \lambda z_H
\]

[Goodman 1996]. Additionally, amplitude-only Fourier holograms require approximately 4 times the desired angular resolution of the 3D object, due to the creation of more than 1 ray by a sinusoid, as shown in Figure 2.
Using these two approximations, the Fresnel hologram’s total resolution can be expressed as the product of the number of Fourier patches \( x_H \) and a desired angular resolution \( \theta_H \) as,

\[
4x_H\theta_H \approx N_H.
\]  

Comparing Eq. 1 with Eq. 4, it is clear that a parallax barrier’s spatio-angular tradeoff is almost identical to the tradeoff of a hologram’s virtual image under a discretized approximation.

### 3.4 Numerical Comparison

Figure 6 demonstrates the operation of each display using either \( 10^4 \) or \( 10^5 \) pixels fit onto a 100mm screen in 1D. The parallax barrier in Figure 6(a) is a successful display setup with \( 10^5 \) pixels that are 0.1 mm wide each, consistent with current slit widths [Harold et al. 2003]. In this example, the screen is split up such that \( x_p = 100 \) and \( \theta_p = 10 \). As pixel sizes shrink, diffraction effects lead the parallax barrier to spread rays across an angle \( \alpha_p = 30^\circ \), washing out image detail (Figure 6(b)). A hologram utilizes the diffraction from \( 10^5 \), 1µm-wide pixels to deliver an image across a 30º viewing angle. From the definition of \( N_H \), this setup can fully reconstruct all parallax information of a 5cm object 30cm away. Discretizing the hologram into 100 Fourier patches will match the spatial resolution of the successful parallax display. Each patch will be a 1000-pixel Fourier transform of the desired image across a 30º angle. From this brief and simplified analysis, two conclusions should be clear:

1. As pixel sizes decrease for a fixed display size, virtual image sharpness and viewing angle conditions improve for a holographic display, while sharpness and light efficiency worsen for a conventional parallax barrier display.
2. Discretizing a hologram into spatial and angular content presents a resolution tradeoff, directly analogous to the space–angle tradeoff in parallax barrier displays.

### 3.5 Experimental Population of Phase Space

A common method of visualizing the spatio-angular tradeoffs of Eq. 1 and Eq. 4 is with a plot of area in space-angle coordinates, or “phase space”. In graphics-based geometric optics, which assumes light is incoherent, simultaneously viewing a ray’s position and angle is often presented as a light field [Levoy and Hanrahan 1996]. The Wigner distribution provides an equally intuitive and closely related plot for coherent wavefront of light [Zhang and Levoy 2009; Testorf et al. 2009]. These related phase space representations will receive a precise definition in Section 4. A schematic 2D phase space plot of the 1D spatial and angular resolution (i.e., space–bandwidth) of both a parallax barrier’s light field and hologram’s Wigner distribution are in Figure 7(a), displaying two transformations:

- **Hologram to light field** A hologram’s Wigner distribution can populate a light field after being downsampled. To demonstrate, images of a recorded hologram’s virtual image from distinct viewing angles are used to populate the horizontal slices of a light field (Figure 7(b)-(c)). The light field is downsampled due to the finite extent of the camera aperture, which averages values across the angle the aperture subsumes, \( \Delta \theta_H \). The light field’s downsampled slices can directly populate horizontal slices of a parallax barrier’s light field, which has a much lower angular resolution (i.e., \( \theta_p = \theta_H/100 \)) in the example in Section 3.4. In the example in Figure 7(b), the 100cm² teacup hologram is imaged from 20x20 discrete positions across a 120º window at 60cm distance with a 50mm f/2.8 lens to populate its geometric light field, with \( \Delta \theta_H = 2.3^\circ \) and \( \Delta \theta_p = 6^\circ \). In Figure 7(c), the same setup is used for the 1100cm² holographic stereogram “The Bartlett Head.” It’s light field is used to generate a point cloud, which is common in graphics literature but offers a new method of interacting with and preserving film-based holograms.

- **Light field to hologram** A light field can be transferred to an approximate holographic representation through the process of discretization. A demonstration using images to create different patches of a stereogram is in [Halle 1994]. Creating a Fresnel hologram’s exact Wigner distribution is much more difficult due to the finite angular sampling \( \Delta \theta_H \) of any sampling instrument (e.g., a camera lens or an eye). Any instrument will introduce averaging over the hologram’s high resolution space-bandwidth details. This averaging effect is equivalent to the finite convolution window required to generate a light field from a Wigner distribution discussed by [Zhang and Levoy 2009]. Typically, the complete phase space representation of a coherent source is estimated using alternative techniques like phase space tomography [Raymer et al. 1994]. To conclude, a light field can approximate a hologram’s phase space function through a discrete mapping process, offering both computational efficiency and a framework to transfer real-world imagery to diffractive display. Future 3D display formats, such as the hybrid display in Section 5, will take advantage of these close links.

### 4 Algebraic Limitations of 3D Displays

Although it is clear that the discrete spatial and angular content of a parallax barrier and hologram can each be easily visualized in phase space, it may not be clear how the incoherent light field and...
coherent Wigner representations are related. In this section, a closer look at these two phase space functions will reveal that the space of 3D light distributions from both a parallax barrier and from a single holographic mask have an algebraic rank-1 limitation.

### 4.1 Rank-1 Geometric Light Field

Before turning to the less familiar description of a hologram in phase space, we will first demonstrate the light field produced by a parallax barrier is rank-1. Still thinking in 1D, it is well known that a joint position \((x)\) and angle \((\theta)\) phase space of all rays is equivalent to a light field parameterization, \(l(x, \theta)\), of a ray passing through two parallel planes \(s_1\) and \(m_2\). Light field propagation is represented as a shear, ray values are always positive, and we will assume rays cannot bend (i.e. diffract) upon interference with a mask. For the simplified parallax barrier in Figure 9(a), the initial screen of pixels creates a light field \(l_1(x, \theta)\) that is constant with broad extent along \(\theta\), \(l_1(x, \theta)\) then shears with free space propagation to become \(l_2(x, \theta)\) directly before the plane of amplitude modulating slits. Here, the slit mask \(m_2(x)\) will either block or allow rays through, defining the light field on the other side of the slits through multiplication, \(l_3(x, \theta) = l_2(x, \theta)m_2(x)\). The light field \(l_3\) then shears across a large distance, represented by a 90° phase space rotation, to a viewer’s eye.

The parallax barrier’s light field can be re-parameterized into an outer–product format by noting \(l_1\) and \(l_2\) are related through the shear relationship,

\[
l_2(x, \theta) = l_1(x-d\theta, \theta),
\]

where \(d\) is the separation between \(s_1\) and \(m_2\). This yields an expression for the output light field \(l_3(x, \theta)\) as a product of two functions,

\[
l_3(x, \theta) = l_1(x-d\theta) m_2(x) = s_1(x-d\theta) m_2(x).
\]

Here, we have replaced \(l_1(x, \theta)\) with \(s_1(x)\), since the initial light field generated by the screen \(s_1(x)\) offers no initial control over the \(\theta\) dimension. Plotting all rays in \(l_3\) in terms of their initial screen coordinate \(s_1(x)\) and mask coordinate \(m_2(x)\) clarify this decomposition (Figure 8(b)). The generated light field \(l_3(x, \theta)\) lies at a 45° angle, with discrete lines of angular content \(\theta\) representing rays at different constant angles. With control over only two amplitude-modulating planes in a parallax barrier, we see that the best rotated light field the display can generate will be the product of two real discrete vectors:

\[
m_1(x)s_2(x)^T = [LF_{45^\circ}]
\]

In other words, parallax barriers are restricted to display rank-1 light fields. Any light field one wishes to display that is not rank-1 will be under-sampled or presented as an aliased image. A specific consequence of parallax barrier displays is low light efficiency: to map a pixel to a desired direction, all other rays are blocked. This significant light attenuation may not be optimal, and recent attempts have been made to improve it [Lanman et al. 2010].

### 4.2 Rank-1 Holographic Light Field

As displays reach resolutions within one order of magnitude of light’s wavelength, the incoherent light field must be transformed into a framework that obeys physical optics. In other words, the assumption that rays cannot bend (i.e., diffract) at a thin screen, like a parallax barrier slit, is not valid at wavelength-scales. As noted above, the Wigner distribution is a direct analogy of the geometric-based light field for coherent light, and includes its diffraction effects [Testorf et al. 2009]. In the limit of a very small wavelength,
The coherent light field after passing through a holographic screen with transmission function $s_1(x)$ is simply the Wigner distribution of $s_1$:

$$W(x, u) = \int s_1(x + x') s_1^*(x - x') e^{-2\pi i x' u} du.$$  \hspace{1cm} (9)

Referring to the example plot in Figure 9(b), it is clear $W(x, u)$ exhibits similar transformation properties as a geometric light field. Free-space propagation of $W(x, u)$ is a shear along the $x$-axis. Furthermore, although $W(x, u)$ can include negative values, projection along the $u$-axis still yields the intensity of light at any plane.

As with the parallax barrier example, we can also transform a hologram’s coherent light field into a space where its limited display capabilities becomes clear (Figure 10(a)). This limitation is implicit in the definition of $W(x, u)$ in Eq. 9, which only relies on the 1D complex screen function $s_1(x)$. We can recover the 1D function $s_1(x)$, up to a constant phase factor [Testorf et al. 2009], by performing 3 transformation operations on the 2D $W(x, u)$. First, a 1D inverse-Fourier transform on $W(x, u)$ is performed along the $u$-axis to yield the expression,

$$F_u^{-1}[W(x, u)] = s_1(x + x') s_1^*(x - x') e^{-2\pi x' u} du.$$  \hspace{1cm} (10)

where $F_u^{-1}$ represents an inverse Fourier transform. The next two operations rotate the expression in Eq. 10 by $45^\circ$, then re-scale the $x_2$-axis by two. This is equivalent to shifting from the center-difference coordinates $(x, x')$ to the two independent coordinates along the mask $(x_1, x_2)$:

$$R_{45} \left[ D \left[ s_1(x + x') S_1^*(x - x') \right] \right] \Rightarrow s_1(x_1) s_1^*(x_2)$$  \hspace{1cm} (11)

The function $s_1(x_1) s_1^*(x_2)$ is often called the mutual intensity function, and describes the statistical correlation between any two points on a wave, or here a holographic screen. Eq. 11 is also a rank-1 representation assuming coherent light [Ozakats et al. 2002]. The rank-1 limitation of a holographic light field manifests itself in effects like speckle and out-of-focus noise [Dorsch et al. 1994]. Intuitively, the constraint arises from the ability to define a wavefront or very large pixels, the Wigner approaches a radiance functions, or rays [Accardi and Wornell 2009]. The key difference between the Wigner distribution and a light field is its replacement of a ray’s angular variable $\theta$ with a wave’s spatial frequency value $u$. Like the decomposition of a light field into many rays, a coherent wavefront can be decomposed into a set of plane waves with different spatial frequencies [Goodman 1996]. Each spatial frequency is proportional to the angle $\theta$ at which each decomposed plane wave propagates:

$$u = \frac{\sin \theta}{\lambda} \approx \frac{\theta}{\lambda}.$$  \hspace{1cm} (8)

where the approximation applies at small diffraction angles. As the 2D Wigner distribution $W(x, u)$ is directly analogous to the light field $I(x, \theta)$ under the assumption of coherent light, we will refer to it as a “coherent light field”. As display pixels approach the order of light’s wavelength, it will be useful to replace rays with localized plane waves that can describe diffraction, and replace $\theta$ with $u$. 

Figure 9: Incoherent and coherent light field representations of (a) a parallax barrier and (b) holographic display to create an image of two points at different depths. The simplified parallax barrier uses 4 pixels to generate an initial light field with constant-$\theta$, which shears to the mask $m_2$ and is attenuated. A viewer sees the rotated version of this attenuated light field. Greater angular separation ($\Delta r$ vs. $\Delta b$) maps to greater parallax disparity when viewing, indicating proximity. A holographic display will use two Fresnel zone plates to create two points in space. The Wigner distribution from this mask will go through identical shears and rotations to a viewer to offer the same disparity. The 2D parallax barrier pattern (c) and holographic pattern (d) for $s_1$ both grow radially, generated with values from the Section 3.4’s numerical example assuming $F_1=10\text{mm}$, $F_2=20\text{mm}$.

Figure 10: (a) The phase space diagram for a 2-slit hologram is a coherent Wigner distribution, which can always be represented by a rank-1 function after applying Eq. 10–11. (b) The same phase space diagram for the two slits is rank-2 assuming multiplexing of the coherent source. For example, each slit could be illuminated by a different laser (spatial multiplexing), or at different moments in time (temporal multiplexing).
at any depth plane through the propagation equation, as long as the
wave’s equation \( s_1(x) \) is fully defined at any earlier plane. The 3-
step process of Eq. 10–11 is applied to a coherent Wigner distribu-
tion of 2 slits, leading to a rank-1 mutual intensity in Figure 10(a).

Connecting Limitations A demonstration of the equivalence of
a rank-1 coherent and incoherent light field as two planes approach
a single plane, based upon the transport-of-intensity equation, is in
the supplementary material. In general, a fully incoherent ray-based
3D display and a fully coherent wave-based 3D display are both
limited in the light fields they generate, with the most flexibility
available somewhere between, achievable through multiplexing.

5 Towards a Hybrid Display
This section describes a hybrid parallax barrier-hologram setup that
delivers unique 4D light fields into different viewing directions
(Figure 11). The display replaces the interlaced images of a paral-
lax barrier with high-resolution segments of separate Fresnel holo-
grams to create depth and angle variant imagery. It takes advantage
of the shared spatio-angular relationship between in Section 3 and overcomes the rank-constraints determined in Section 4.

Instead of serving as a final display solution, however, it is simply
intended as a novel example of 3D display development to encour-
gage others to examine 3D display from an algebraic-rank viewpoint.

Multiplexing and Discretization Without considering algebraic
rank, related work often investigates various multiplexing schemes
to improve 3D display, as noted in Section 3.3. For example, tem-
porally multiplexing a parallax barrier will generate \( n \) independent
light fields within the flicker-fusion rate of a viewer’s eye, allow-
ing the displayed light field to be a rank-\( n \) function [Dodgson et al.
2000; Lanman et al. 2010]. Multiplexing can also improve the rank
of a hologram’s coherent light field. The stereographic and holog-
ically discretized holograms from Section 3.3 are multiplexed spatially, while
advanced displays like the HoloTV [Hilaire et al. 1991] are multi-
plexed temporally. Multiplexing is also closely related to the illu-
minations partial coherence state [Ozaktas et al. 2002]. An example of multiplexing’s benefit is shown for the two-slit example in Fig-
ure 10(b), where either spatial or temporal multiplexing can remove
the slit’s interference fringes, extending the light field’s algebraic
constraint from rank-1 to rank-2.

Mixed Spatio-angular Tradeoff The proposed hybrid display
utilizes angular multiplexing to generate \( \theta_p \) coherent light fields
that change dramatically with viewing angle. Angular multiplex-
ing is common in holographic data storage [Mok 1993]. It was ex-
tended to CGH’s that create 2D images [Gerke and Fiestun 2010],
but only with volumetric holograms that are not conducive towards
large-format display. Just like the images of a parallax barrier, this
display architecture effectively interlaces \( \theta_p \) holograms, each with \( N_H \) pixels, beneath a set of \( x_p \) slits. However, instead of each slit corresponding to \( \theta_p \) pixels of discrete angular content, each slit contains a set of \( \theta_p \) \( x_p \) \( t_H \) pixels beneath it, where \( t_f \) and \( t_r \) are the pixel and slit width, respectively. A new mixed spatio-angular tradeoff equation for a total number of screen pixels \( N \) can now be expressed as,

\[
N = \theta_p \theta_p x_H = \theta_p x_p t_H,
\]

highlighting the inter-relation of all 4 spatio-angular variables of
Section 3 in this new display form.

Increased Algebraic Rank This hybrid display example in-
creases the algebraic rank of both Eq. 7 and Eq. 11. Examining the coherent light field restriction in Eq. 11, since \( \theta_p \) different rank-
1 holographic images are simultaneously multiplexed onto a sin-
gle display, a rank-\( \theta_p \) coherent light field is created (rank-3 in Fig-
ure 12(a)). Determining the algebraic rank associated with the in-
coherent light field analysis of Section 4.1 is slightly more chal-
 lenging, since its diffraction-free assumption is no longer valid. In-
tuitively, a ray passing through the screen plane \( s_1 \) will mix (i.e.,
diffact) with neighboring pixels as they pass through the mask \( \mu_2 \). Diffraction of a ray across a plane can be represented as a convolu-
tion [Bastiaans 1979]. In an optimal setup, parameters will be chos-
en such that the holographic screen diffracts light across the entire
slit width, leading to an incoherent light field with a maximum rank
established by the number of holographic pixels beneath each slit,
set by \( r_p / t_H \). Figure 12(b) provides an example of the incoherent
light field’s transform space assuming each strip of hologram con-
tains 3 pixels (i.e., \( r_p / t_H = 3 \)), leading to a rank-3 display, just as
determined by coherent rank-analysis of the hybrid display. How-
ever, each space offering an equal rank limitation of a given display
is not guaranteed.

Experiment and Simulation Figure 11(a) offers a simulation of a
proposed 6D display, which can direct different light fields in
different directions, with parameters listed in the caption. Each
2048\(^2 \) pixel hologram is generated using the desired input images in
Figure 11(b) from a well-known phase-retrieval procedure [Dorsch
et al. 1994]. Phase retrieval determines a holographic pattern to
generate completely different desired images at 2 or more depths.
Here only 2 depths are modeled, for simplicity. The mean-squared
error between the desired and modeled 3D images is .0194. The
modeled display containing 6144\(^2 \) \( t_H \) pixels is beyond current
fabrication techniques. However, a lower resolution film-based
example to generate 2D holographic images is presented in Figure 11(c) as a proof-of-concept.

**Limitations** This hybrid display is primarily offered as an example of similarity between the two display forms and to compare their rank-limitations. Drawbacks include the unnecessary creation of multiple image orders common to all amplitude-only based hologram generation. Additional orders are typically blocked. Light efficiency also decreases with the addition of slits, and multiple illumination sources are needed for simultaneous viewing. Furthermore, cross-talk between neighboring interlaced holograms must be limited through padding or baffling in an experimental implementation. However, this novel form of display is one step towards the merger of two methods of presenting 3D images that, as pixels continue to scale down, are bound to intersect.

6 Conclusion and Future Work

This paper has offered several key insights to facilitate the adoption of diffractive optics into the displays of the future. First, discretizing a hologram allows for a nearly identical spatio-angular treatment as a parallax barrier, as geometric light fields can provide an approximate method of hologram creation, and vice-versa. Second, a phase space analysis of both display forms demonstrates a clear algebraic restriction on their 3D light distributions, suggesting alternative display formats may be superior. Finally, multiplexing can help overcome these restrictions, although considering the problem from an algebraic standpoint instead of multiplexing offers an entire mathematical toolset to promote future light field analysis of incoherent, coherent and partially coherent display.

References


