Macroscopic Interferometry with Electrons, not Photons

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An interferometric 3D camera for multipath imaging for meter scale scenes

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❑ Key Contribution

- **Theory**
  - Formal link between electronic interferometry (like ToF cameras) and optical interferometry (like OCT).
  - Bound on pathlength difference of multipath reflections.
- **Practice**
  - Improved 3D sensing with ToF cameras by incorporating theoretical elements from OCT.

❑ Concept

ToF cameras emit a sinusoidal illumination signal:

$$i(t) = \alpha \cos(2\pi f_{\mathrm{M}} t) + \beta_i$$

CMOS sensor correlates returned signal with reference:

$$c(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} o(t) r(t+\tau) dt = \frac{\alpha}{2} \cos(2\pi f_{\mathrm{M}} \tau + \varphi).$$

Phase sampling leads to closed-form solution for phase:

$$\varphi = \arctan\left(\frac{c(\tau_4) - c(\tau_2)}{c(\tau_1) - c(\tau_3)}\right)$$

But assumes only one phase is received. Manipulation of (2) yields:

$$c(\tau, f_{\mathrm{M}}) = \frac{\alpha}{2} \cos\left(2\pi f_{\mathrm{M}} \tau + \frac{2\pi z}{c} f_{\mathrm{M}}\right) + \beta$$

We propose fixing phase, yet sweeping the frequency such that:

$$c(\tau = 0, f_{\mathrm{M}}) = c(f_{\mathrm{M}}) = \frac{\alpha}{2} \cos\left(\frac{2\pi z}{c} f_{\mathrm{M}}\right) + \beta$$

Compute FFT to recover multiple pathlengths:

$$c(f_{\mathrm{M}}) = \frac{1}{2}\left(\sum_{l=1}^{K} \alpha_l \cos\left(\frac{2\pi z_l}{c} f_{\mathrm{M}}\right)\right) + \beta.$$

$$\mathcal{F}[c(f_{\mathrm{M}})](\kappa) \propto \delta(\kappa) + \sum_{l=1}^{K} \alpha_l \delta\left(\kappa \pm \frac{2\pi z_l}{c}\right)$$

Our work evaluates this approach empirically and theoretically.

❑ Swept Frequency Electronic Interferometry

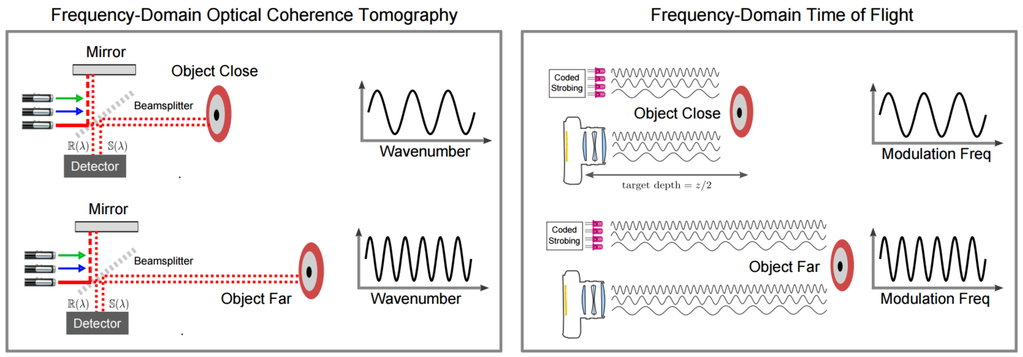


Figure 1. Repurposing the microsocopic technique of Frequency-Domain OCT to a macroscopic technique, Frequency ToF. Short optical paths lead to a lower frequency in the domain of modulation frequency.

❑ Prototype Camera

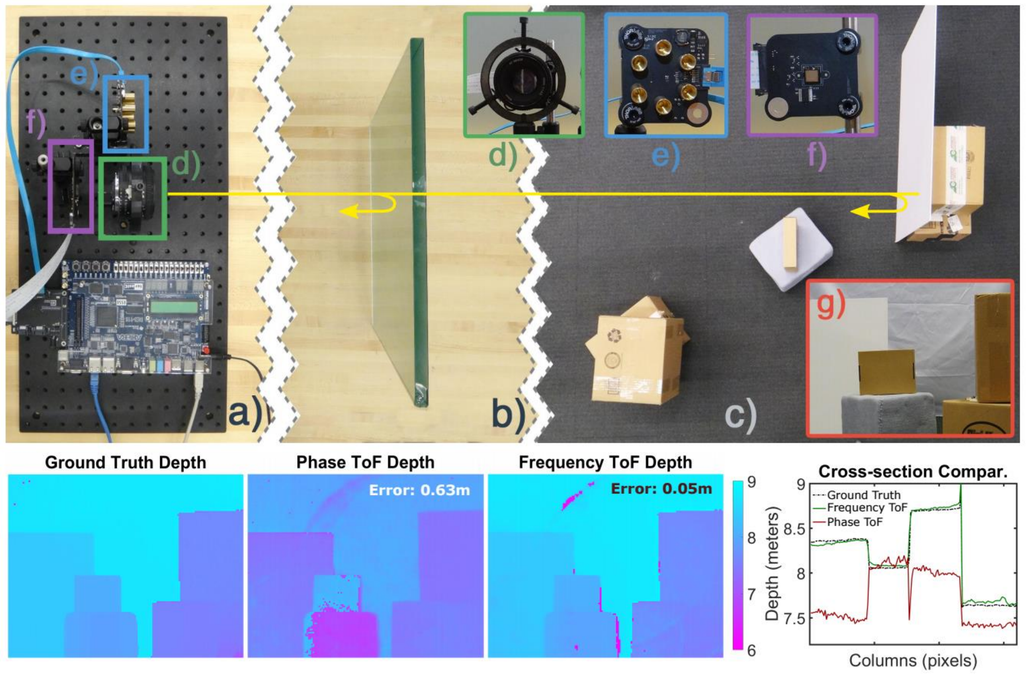
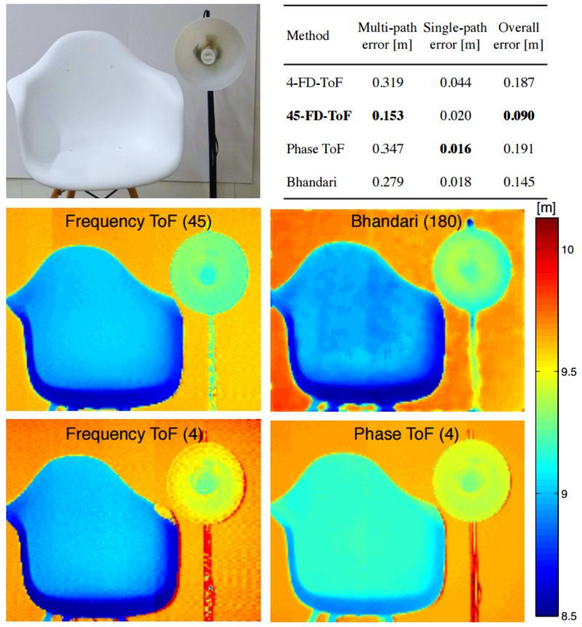


Figure 2. Ranging of objects through a window capturing multiple electronic phases. The glass is 4.75m in front of sensor plane and first box is 7.4m in front. Note the prototype camera with zoomed in lens (d), lasers (e) and sensor (f).

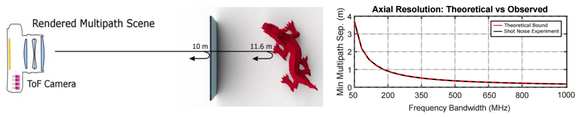
❑ Complex Objects



Method	Multi-path error [m]	Single-path error [m]	Overall error [m]
4-FD-ToF	0.319	0.044	0.187
45-FD-ToF	**0.153**	0.020	**0.090**
Phase ToF	0.347	**0.016**	0.191
Bhandari	0.279	0.018	0.145

Fig. 3. Concave objects with interreflections

❑ Phase Separation Bound



Proposition 1: *Multi-path interference can be resolved in Frequency-Domain ToF if the optical path-length between any two reflections is less than:*

$$\Delta z \approx 0.6c/\Delta f_{\mathrm{M}}.$$

Theoretical bound on reflection separation wrt. distance and bandwidth.

❑ References

- Huang, 1991, Science
- Yuan, 2001, PhD Thesis Univ. Siegen
- Heide et al., 2013, SIGGRAPH
- Bhandari et al., 2014 Optics Letters
- Gupta et al., 2015, ACM Trans. Graphics
- Whyte et al., 2015, Applied Optics
- Holloway et al. 2016, IEEE Trans. Comp. Img

❑ **Take-home message:** Synergy between electronic and optical interferometry especially in the macroscopic domain.