Macroscopic Interferometry

Achuta Kadambi, MIT Media Lab

Joint work with Jamie Schiel and Ramesh Raskar
Computer Vision/Graphics
Computer Vision/Graphics

Real-Life

Electric field only shown

700 nanometer wavelength
Computer Vision/Graphics

Real-Life

Microscopic wave phenomena → Macro scenes

Electric field only shown

700 nanometer wavelength
Multi-depth 3D Camera
Multi-depth 3D Camera

10 meter scene
Multi-depth 3D Camera

Performance competitive low SNR

10 meter scene
Previous Work

Polarized 3D: Depth Sensing with Polarization Cues

Achuta Kadambi\textsuperscript{1}  Vage Taamazyan\textsuperscript{2}  Boxin Shi\textsuperscript{3}  Ramesh Raskar\textsuperscript{1}

\textsuperscript{1}MIT Media Lab  \textsuperscript{2}Skoltech  \textsuperscript{3}SUTD

ICCV 2015, Santiago Chile

ICCV15: Uses wave phenomena of polarization to enhance 3D shape
Previous Work

Polarized 3D: Depth Sensing with Polarization Cues

Achuta Kadambi¹ Vage Taamazyan² Boxin Shi³ Ramesh Raskar¹

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ICCV 2015, Santiago Chile

ICCV15: Uses wave phenomena of polarization to enhance 3D shape
Today: Use wave phenomena to create multi-depth 3D
Scene

A

B
$I = I_A$
$I = I_B$
The diagram illustrates a scene with two light bulbs, labeled A and B. The light intensity \( I \) is given by the sum of the individual intensities \( I_A \) and \( I_B \):

\[
I = I_A + I_B
\]
\[ I = I_A + I_B + 2\sqrt{I_A I_B \cos \Delta \phi} \]

Interference Term

Scene
$I = I_A + I_B + 2\sqrt{I_A I_B \cos \Delta \phi}$

Interference graphic from sciencetalentent.dk
Interferometry

*Intentionally superimposing light to get scene clues*

Figure from ligo.Caltech.edu
Interferometry uses a reference to probe the scene

Reference Signal

Sample Signal

Phase Delay
Tiny Wavelength Good for Microscopic Scenes

Reference Signal

Sample Signal

700 nanometer wavelength

Phase Delay
Tiny Wavelength Good for Microscopic Scenes

Reference Signal

Sample Signal

Phase Delay

700 nanometer wavelength

Corneal OCT

Corneal OCT from Carl Zeiss Visante®
Tiny Wavelength Good for Microscopic Scenes

Reference Signal

Sample Signal

700 nanometer wavelength

Phase Delay

Corneal OCT from Carl Zeiss Visante®
Measure sample deviations less than the size of a proton

LIGO Project to Detect Grav Waves
Smaller the measurement .. Bigger the instrument.

LIGO Project to Detect Grav Waves

Measure sample deviations less than the size of a proton

.. But kilometers in size
Vision goal: small device, large scene
Vision goal: small device, large scene

Optical Phase

700 nanometer wavelength
Vision goal: small device, large scene

Optical Phase

Electronic Phase

700 nanometer wavelength

Kinect uses electronic phase
Terminology

**Primal-Domain:** Original Frame a Signal is Sampled In

**Dual-Domain:** Frequency transform with respect to Primal

**Phase ToF:** existing Kinect-style ToF cameras
Phase ToF (Kinect)
Phase ToF (Kinect)

\[ z = \frac{c \phi}{2\pi f_M} \]
How the Phase ToF (Kinect) Works

\[
dt = 0.0001; \\
N = 20000; \\
t = 0:dt:(N-1)*dt; \\
phi1 = pi/3; phi2 = 0; \\
s1 = sin(2*pi*1*t + phi1); \\
s2 = sin(2*pi*1*t + phi2); \\
xcFFT = conj(fft(s1)) .* fft(s2); \\
[~, fundamental_idx] = max(xcFFT); \\
phase_difference = angle(xcFFT(fundamental_idx));
\]
Phase ToF Refer. 
\[ r(t) = \cos(f_M t) \]

Phase ToF Sample 
\[ s(t) = \cos(f_M t + \phi) \]

Primal-domain is \( \tau \)

For clarity, all constants removed
Block Diagram of Phase ToF

\[ r(t) = \cos(f_M t) \]

\[ s(t) = \cos(f_M t + \phi) \]

\[ c(\tau) = \cos(f_M \tau + \phi) \]

For clarity, all constants removed
Compute FFT wrt. $\tau$ & Take angle of fundam.

Phase ToF Xcorr

$c(\tau) = \cos(f_M \tau + \varphi)$

Primal-domain is $\backslash \tau$

Phase ToF Refer.

$r(t) = \cos(f_M t)$

Phase ToF Sample

$s(t) = \cos(f_M t + \varphi)$

For clarity, all constants removed
Problems with Phase ToF

**Multipath Interference**

Reflections of two or more sine waves come back and mix

**Phase Wrapping**

Phase is periodic – long-range depths will wrap

**Low SNR**

Calculating phase requires multiple steps (prop. of errors)
Computational ToF Imaging

By count of recent papers, appears to be a hot area in comp. photo

[Godbaz et al. 2008]
[Heide and Hullin et al. 2013]
[Kadambi et al. 2013]
[Bhandari et al. 2014]
[Jayasuriya et al. 2015]
[Gkioulekas et al. 2015]
[Tsai et al. 2016]

Find more details at ICCV 2015 course: www.media.mit.edu/~achoo/iccvtoftutorial/

[Su et al. 2016] -- @ Wed AM posters
[Shreshtha et al. 2016] -- @ next month’s SIGGRAPH
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Our Contribution: change primal-domain
Changing the Primal Domain

**Phase ToF Refer.**
\[ r(t) = \cos(f_M t) \]

**Phase ToF Sample**
\[ s(t) = \cos(f_M t + \varphi) \]

\[ \times \]

**Phase ToF Xcorr**
\[ c(\tau) = \cos(f_M \tau + \varphi) \]

**Compute FFT wrt. \( \tau \)**

Primal-domain is \( \tau \)

For clarity, all constants removed
Changing the Primal Domain

Phase ToF Xcorr

\[ c(\tau) = \cos(f_M \tau + \varphi) \]

For clarity, all constants removed
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Changing the Primal Domain

Phase ToF Xcorr

\[ c(\tau) = \cos(f_M \tau + \phi) \]

\[ z = \frac{c\phi}{2\pi f_M} \]

For clarity, all constants removed
Changing the Primal Domain

Phase ToF Xcorr

\[ c(\tau) = \cos(f_M \tau + \varphi) \]

\[ z = \frac{c \varphi}{2\pi f_M} \]

\[ c(\tau) = \cos\left(f_M \tau + \frac{2\pi z}{c} f_M\right) \]
Changing the Primal Domain

Phase ToF Xcorr

\[ c(\tau) = \cos(f_M \tau + \phi) \]

\[ c(\tau) = \cos\left(f_M \tau + \frac{2\pi z}{c} f_M\right) \]

\[ c(\tau, f_M) = \cos\left(f_M \tau + \frac{2\pi z}{c} f_M\right) \]

For clarity, all constants removed
Changing the Primal Domain

Phase ToF Xcorr

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\[ c(\tau, f_M) = \cos\left(f_M \tau + \frac{2\pi z}{c} f_M\right) \]

\[ c(\tau = 0, f_M) = \cos\left(f_M 0 + \frac{2\pi z}{c} f_M\right) \]

For clarity, all constants removed
Changing the Primal Domain

**Phase ToF Xcorr**

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\[ z = \frac{c \phi}{2\pi f_M} \]

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Frequency ToF Xcorr

\[ z = \frac{c\varphi}{2\pi f_M} \]

\[ c(f_M) = \cos\left(\frac{2\pi z}{c} f_M\right) \]

For clarity, all constants removed
Changing the Primal Domain

**Phase ToF Xcorr**

\[
c(\tau) = \cos(f_M \tau + \phi)
\]

\[
z = \frac{c\phi}{2\pi f_M}
\]

**Frequency ToF Xcorr**

\[
c(f_M) = \cos\left(\frac{2\pi z}{c} f_M\right)
\]

- Primal-domain is \(f_M\)
- Dual-domain corresponds to path length.

For clarity, all constants removed
Frequency-Domain Time of Flight
Macroscopic Scenes

Long-range scene

3.4m
Macroscopic Scenes

Long-range scene

Phase ToF

3.4m
Macroscopic Scenes

Long-range scene

Phase ToF

Frequency-Domain ToF

3.4m
Need only 4 frequencies to recover both reflections
Multi-depth 3D Camera

Need only 4 frequencies to recover both reflections
Multi-depth 3D Camera

Need only 4 frequencies to recover both reflections
Resolving Multipath Interference

Scene

<table>
<thead>
<tr>
<th>Method</th>
<th>Multi-path error [m]</th>
<th>Single-path error [m]</th>
<th>Overall error [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-FD-ToF</td>
<td>0.319</td>
<td>0.044</td>
<td>0.187</td>
</tr>
<tr>
<td>45-FD-ToF</td>
<td>0.153</td>
<td>0.020</td>
<td>0.090</td>
</tr>
<tr>
<td>Phase ToF</td>
<td>0.347</td>
<td><strong>0.016</strong></td>
<td>0.191</td>
</tr>
<tr>
<td>Bhandari</td>
<td>0.279</td>
<td>0.018</td>
<td>0.145</td>
</tr>
</tbody>
</table>
How close can the reflections be?

**Proposition 1:** Multi-path interference can be resolved in Frequency-Domain ToF if the optical path-length between any two reflections is less than:

\[ \Delta z \approx 0.6c/\Delta f_M. \]
How close can the reflections be?

**Proposition 1:** Multi-path interference can be resolved in Frequency-Domain ToF if the optical path-length between any two reflections is less than:

\[ \Delta z \approx 0.6c/\Delta f_M. \]
Analyzing multipath estimation in low SNR

Multipath Depth Sensing Error

- Frequency ToF (Proposed)
- Phase ToF
- Phase ToF MPI Corrected

Typical Range

SNR (dB)

Mean Abs. Err (dB)

30 dB SNR

15 dB SNR

Frequency ToF [Proposed]
Phase ToF [Kinect]
Phase ToF MPI Corrected [Bhandari 2014]
Conclusion

Formalize link between ToF cameras and optical interferometry

Changed primal-domain for multi-depth, low SNR, long-range 3D

Collaborators

Jamie Schiel
Ramesh Raskar

Acknowledgements:

Ayush Bhandari
Suren Jayasuriya
Vage Taamazyan

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